

Systematic Code Generation for ML Computations based on Multi-Dimensional Homomorphisms

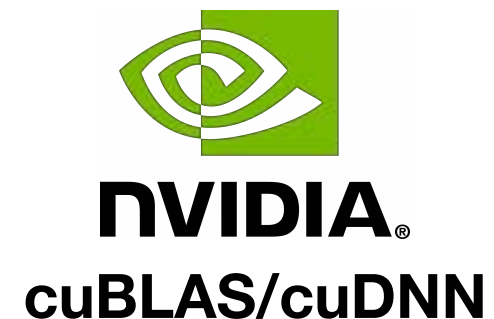
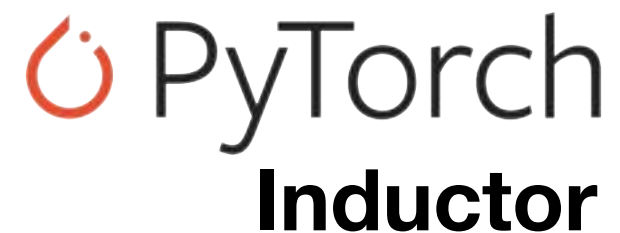
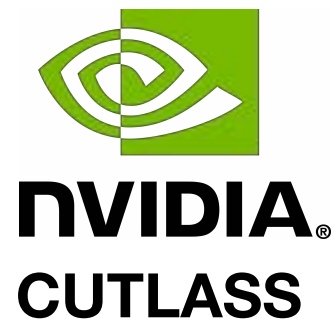
Ari Rasch, Richard Schulze
University of Münster, Germany

Goal of this Talk

Observation:

State-of-the-art **ML compilers/libraries** struggle with **fully automatically** achieving **high performance** for an **extensible set of target architectures**

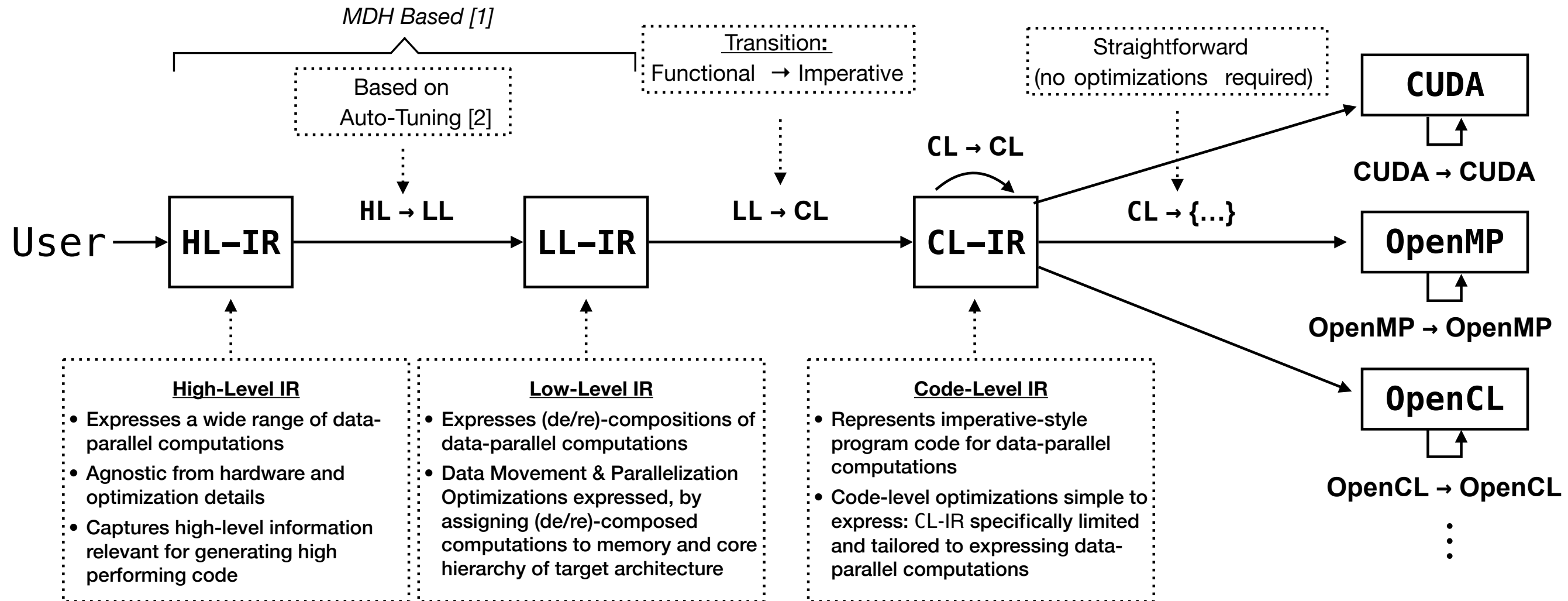
Examples:



We present a systematic **ML code generation process** that **fully automatically** achieves **high performance** across an **extensible set of architectures**

Overview

End-to-end ML Code-Generation approach:



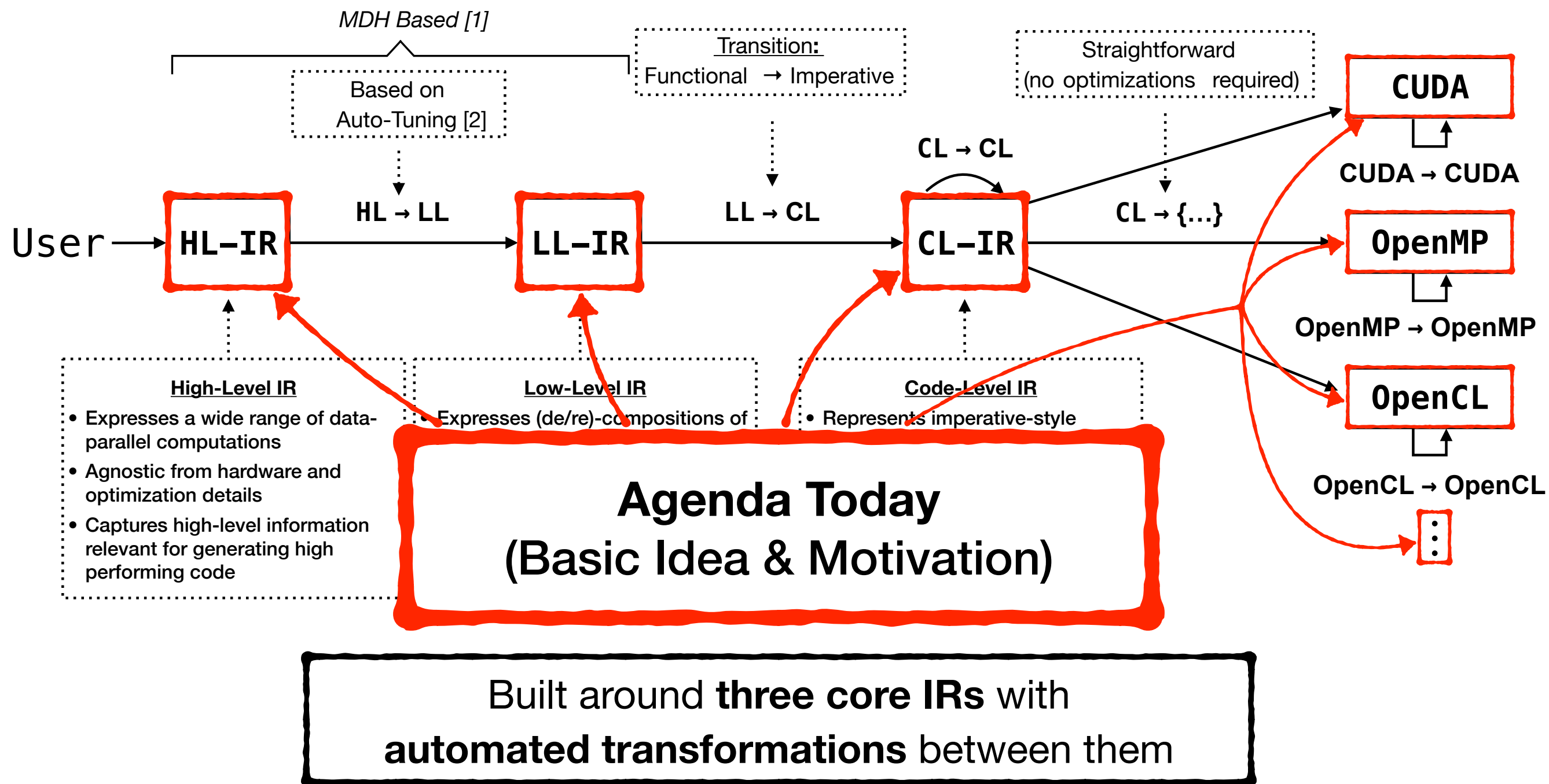
Built around **three core IRs** with **automated transformations** between them

[1] Rasch, “(De/Re)-Composition of Data-Parallel Computations via Multi-Dimensional Homomorphisms”, TOPLAS’24

[2] Rasch, Schulze, Steuwer, Gorlatch, “Efficient Auto-Tuning of Parallel Programs with Interdependent Tuning Parameters via Auto-Tuning Framework (ATF)”, TACO’21

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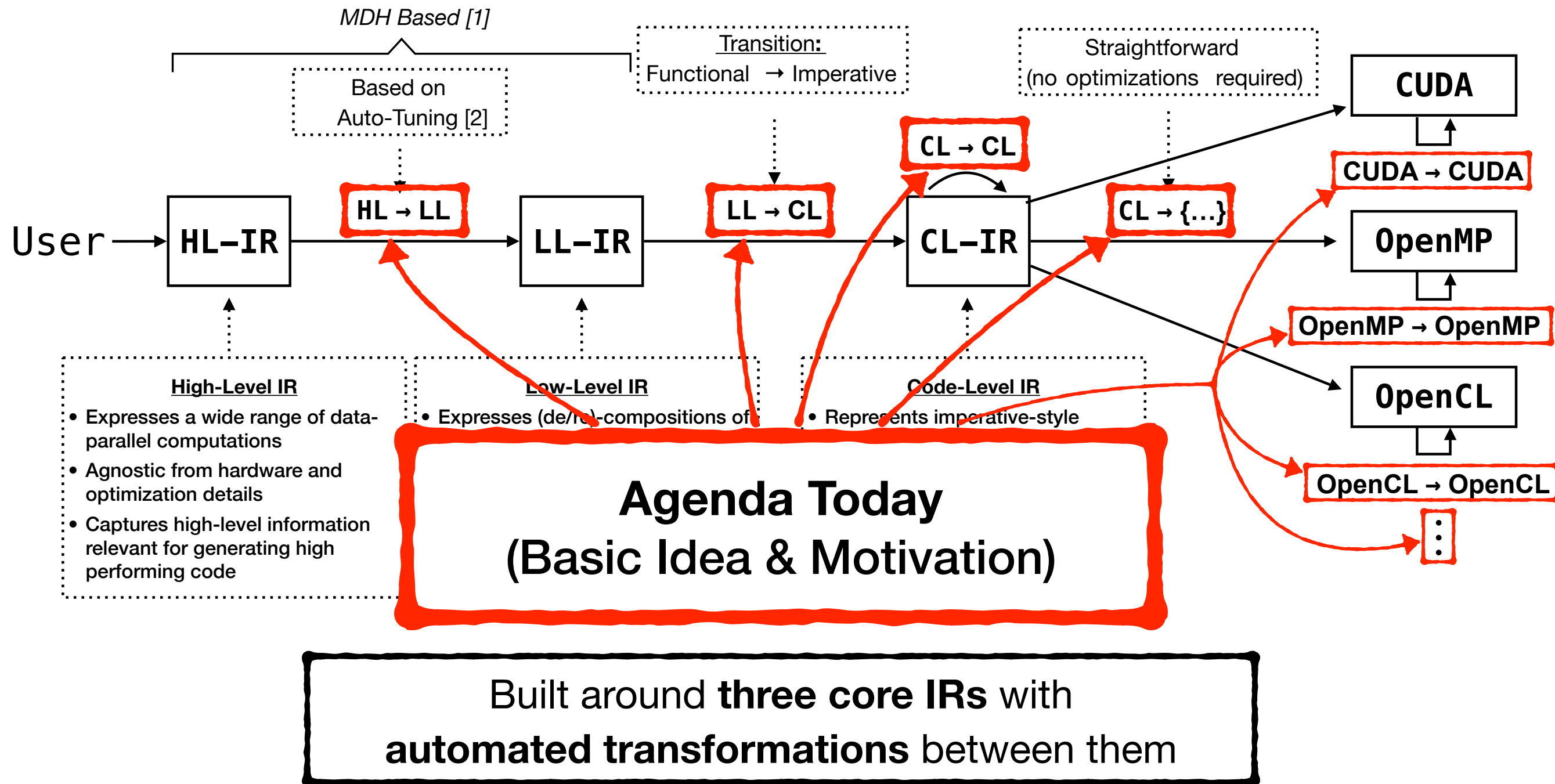


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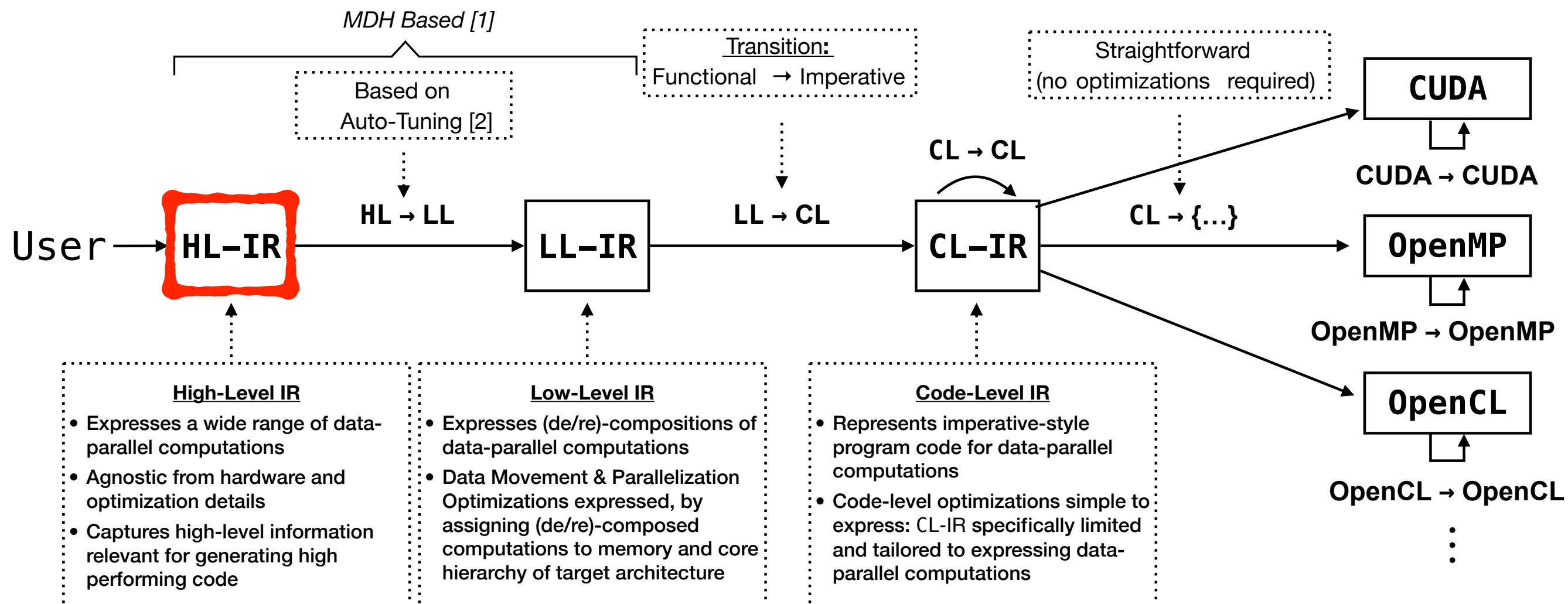


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HL-IR: High-Level IR

Example: MatVec expressed in HL-IR (using its Python DSL representation)

```
def matvec( T: BasicType, I: int, K: int ):
    @mdh( )
    def mdh_matvec():
        return (
            out_view[T]( w = [lambda i,k: (i)] ),
            md_hom[I,K]( mul, ( cc, pw(scalar_plus) ) ),
            inp_view[T,T]( M = [lambda i,k: (i,k)] ,
                           v = [lambda i,k: (k) ] ) )
```



High-Level Representation of MatVec

What is happening here:

- `inp_view` captures the accesses to input data
- `md_hom` expresses the core algebraic computation
- `out_view` captures the accesses to output data

The HL-IR **uniformly expresses**
ML computations,
abstracting away low-level details
while **preserving high-level**
algebraic information

HL-IR: ML Examples

md_hom	f	$\otimes_1, \dots, \otimes_D$
MatMul<F,F>	*	++, ++, +
MatMul<F,T>	*	++, ++, +
MatMul<T,F>	*	++, ++, +
MatMul<T,T>	*	++, ++, +
BatchMatMul<F,F>	*	++, ..., ++, +
⋮	⋮	⋮
BiasAddGrad<NHWC>	id	+, +, +, ++
BiasAddGrad<NCHW>	id	+, ++, +, +
CheckNumerics	$(x) \mapsto (x == \text{NaN})$	$\vee, \dots, \vee$
Sum<0><F>	id	+, ++, ++, ..., ++
Sum<0><T>	id	+, ++, ++, ..., ++
Sum<1><F>	id	++, +, ++, ..., ++
Sum<0,1><F>	id	+, +, ++, ..., ++
⋮	⋮	⋮
Prod<0><F>	id	*, ++, ++, ..., ++
⋮	⋮	⋮
All<0><F>	id	&&, ++, ++, ..., ++
⋮	⋮	⋮

Linear Algebra, Contractions, ...
(Computation Specification)

Views	inp_view		out_view
	I_1	I_2	O
MatMul<F,F>	$(i, j, k) \mapsto (i, k)$	$(i, j, k) \mapsto (k, j)$	$(i, j, k) \mapsto (i, j)$
MatMul<F,T>	$(i, j, k) \mapsto (i, k)$	$(i, j, k) \mapsto (j, k)$	$(i, j, k) \mapsto (i, j)$
MatMul<T,F>	$(i, j, k) \mapsto (k, i)$	$(i, j, k) \mapsto (k, j)$	$(i, j, k) \mapsto (i, j)$
MatMul<T,T>	$(i, j, k) \mapsto (k, i)$	$(i, j, k) \mapsto (j, k)$	$(i, j, k) \mapsto (i, j)$
BatchMatMul<F,F>	$(b_1, \dots, i, j, k) \mapsto (b_1, \dots, i, k)$	$(b_1, \dots, i, j, k) \mapsto (b_1, \dots, k, j)$	$(b_1, \dots, i, j, k) \mapsto (b_1, \dots, i, j)$
⋮	⋮	⋮	⋮
BiasAddGrad<NHWC>	$(n, h, w, c) \mapsto (n, h, w, c)$	/	$(n, h, w, c) \mapsto (n, h, w)$
BiasAddGrad<NCHW>	$(n, c, h, w) \mapsto (n, c, h, w)$	/	$(n, c, h, w) \mapsto (n, h, w)$
CheckNumerics	$(i_1, \dots, i_D) \mapsto (i_1, \dots, i_D)$	/	$(i_1, \dots, i_D) \mapsto ()$
Sum<0><F>	$(i_1, \dots, i_D) \mapsto (i_1, \dots, i_D)$	/	$(i_1, \dots, i_D) \mapsto (i_2, \dots, i_D)$
Sum<0><T>	$(i_1, \dots, i_D) \mapsto (i_1, \dots, i_D)$	/	$(i_1, \dots, i_D) \mapsto (0, i_2, \dots, i_D)$
Sum<1><F>	$(i_1, \dots, i_D) \mapsto (i_1, \dots, i_D)$	/	$(i_1, \dots, i_D) \mapsto (i_1, i_3, \dots, i_D)$
Sum<0,1><F>	$(i_1, \dots, i_D) \mapsto (i_1, \dots, i_D)$	/	$(i_1, \dots, i_D) \mapsto (i_3, \dots, i_D)$
⋮	⋮	⋮	⋮
Prod<0><F>	$(i_1, \dots, i_D) \mapsto (i_1, \dots, i_D)$	/	$(i_1, \dots, i_D) \mapsto (i_2, \dots, i_D)$
⋮	⋮	⋮	⋮
All<0><F>	$(i_1, \dots, i_D) \mapsto (i_1, \dots, i_D)$	/	$(i_1, \dots, i_D) \mapsto (i_2, \dots, i_D)$
⋮	⋮	⋮	⋮

Linear Algebra, Contractions, ... (Data Specification)

md_hom	f	$\otimes_1, \dots, \otimes_D$
Fill	id	++, ..., ++
ExpandDims<0>	id	++, ..., ++
ExpandDims<1>	id	++, ..., ++
ExpandDims<0,1>	id	++, ..., ++
⋮	⋮	⋮
Transpose< σ >	id	++, ..., ++
Exp	exp	++, ..., ++
Mul	*	++, ..., ++
BiasAdd<NHWC>	+	++, ++, ++, ++
BiasAdd<NCHW>	+	++, ++, ++, ++
Range	$(s, d, i) \mapsto (s + d * i)$	++

Point-Wise, Re-Shaping, ...
(Computation Specification)

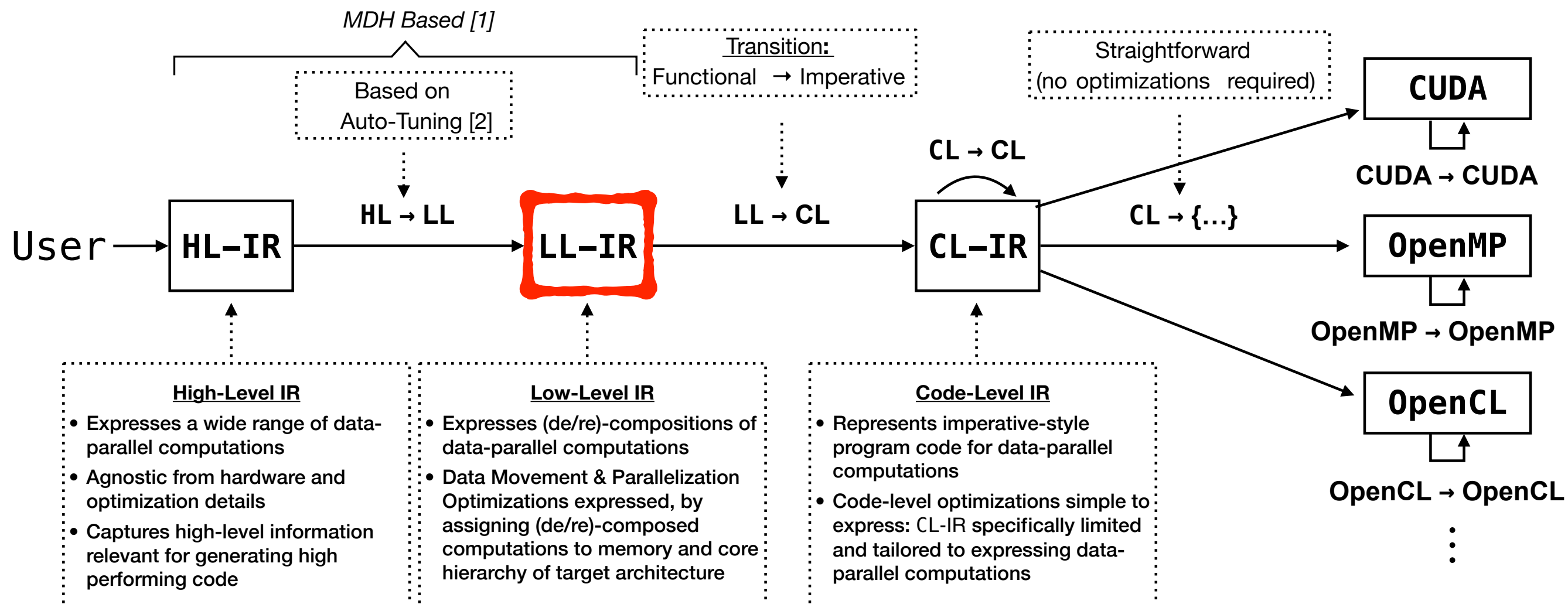
Views	inp_view		out_view
	I_1	I_2	O
Fill	$(i_1, \dots, i_D) \mapsto ()$	/	$(i_1, \dots, i_D) \mapsto (i_1, \dots, i_D)$
ExpandDims<0>	$(i_1, \dots, i_D) \mapsto (i_1, \dots, i_D)$	/	$(i_1, \dots, i_D) \mapsto (0, i_1, i_2, \dots, i_D)$
ExpandDims<0>	$(i_1, \dots, i_D) \mapsto (i_1, \dots, i_D)$	/	$(i_1, \dots, i_D) \mapsto (i_1, 0, i_2, \dots, i_D)$
ExpandDims<0,1>	$(i_1, \dots, i_D) \mapsto (i_1, \dots, i_D)$	/	$(i_1, \dots, i_D) \mapsto (0, 0, i_1, \dots, i_D)$
⋮	⋮	⋮	⋮
Transpose< σ >	$(i_1, \dots, i_D) \mapsto (\sigma(i_1), \dots, \sigma(i_D))$	/	$(i_1, \dots, i_D) \mapsto (i_1, \dots, i_D)$
Exp	$(i_1, \dots, i_D) \mapsto (i_1, \dots, i_D)$	/	$(i_1, \dots, i_D) \mapsto (i_1, \dots, i_D)$
Mul	$(i_1, \dots, i_D) \mapsto (i_1, \dots, i_D)$	$(i_1, \dots, i_D) \mapsto (i_1, \dots, i_D)$	$(i_1, \dots, i_D) \mapsto (i_1, \dots, i_D)$
	$(i_1, \dots, i_D) \mapsto (i_1, \dots, i_D)$	$(i_1, \dots, i_D) \mapsto (i_1, \dots, i_{k-1}, i_{k+1}, \dots, i_D)$	$(i_1, \dots, i_D) \mapsto (i_1, \dots, i_D)$
⋮	⋮	⋮	⋮
BiasAdd<NHWC>	$(n, h, w, c) \mapsto (n, h, w, c)$	$(n, h, w, c) \mapsto (c)$	$(n, h, w, c) \mapsto (n, h, w, c)$
BiasAdd<NCHW>	$(n, c, h, w) \mapsto (n, c, h, w)$	$(n, c, h, w) \mapsto (c)$	$(n, c, h, w) \mapsto (n, c, h, w)$
Range	$(i) \mapsto ()$	$(i) \mapsto ()$	$(i) \mapsto (i)$

Point-Wise, Re-Shaping, ... (Data Specification)

Important **ML kernels** can be expressed **uniformly** in HL-IR

Overview

End-to-end *ML Code-Generation* approach:



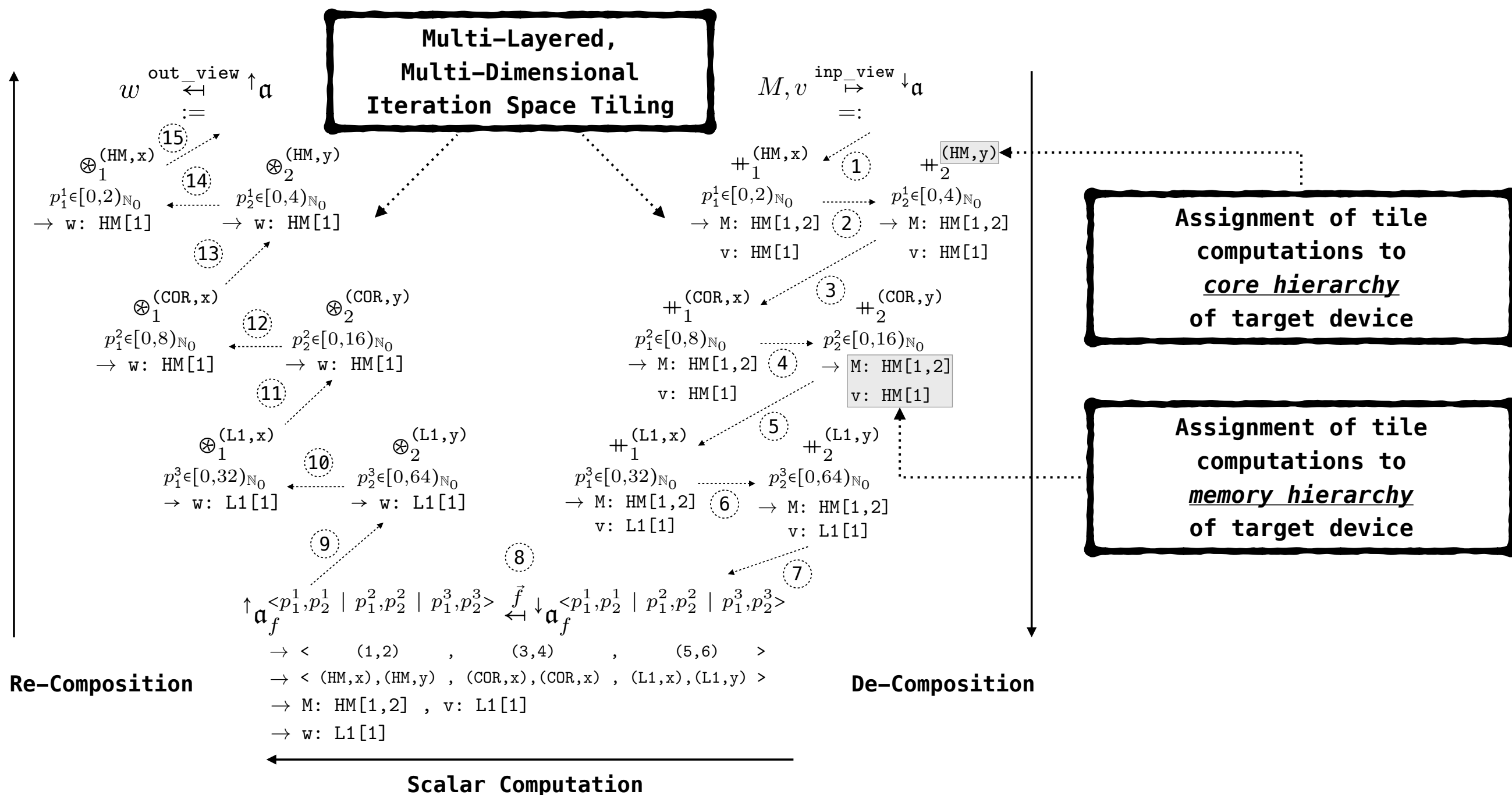
Built around **three core IRs** with **automated transformations** between them

[1] Rasch, “*(De/Re)-Composition of Data-Parallel Computations via Multi-Dimensional Homomorphisms*”, TOPLAS’24

[2] Rasch, Schulze, Steuwer, Gorlatch, “*Efficient Auto-Tuning of Parallel Programs with Interdependent Tuning Parameters via Auto-Tuning Framework (ATF)*”, TACO’21

LL-IR: Low-Level IR

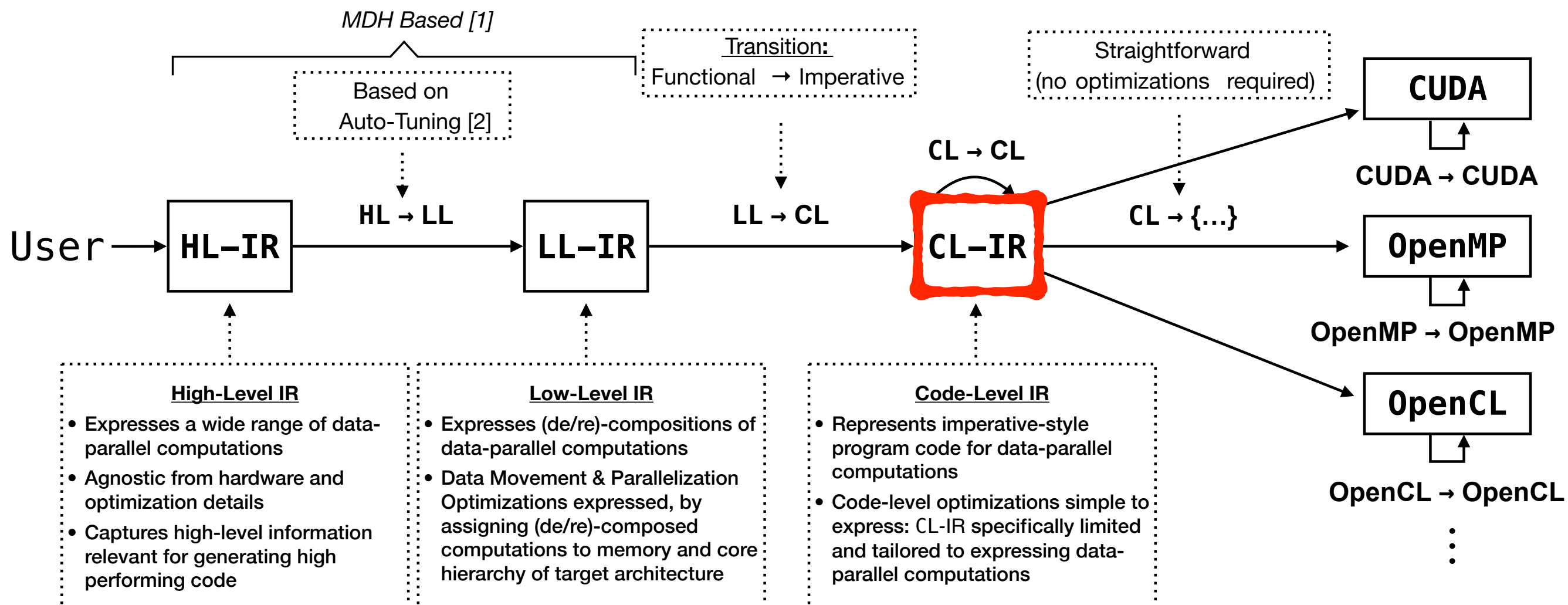
Goal: Expressing optimized *de-composition* and *re-composition* of ML computations



Expresses *parallelization* and *data-movement* optimizations

Overview

End-to-end ML Code-Generation approach:



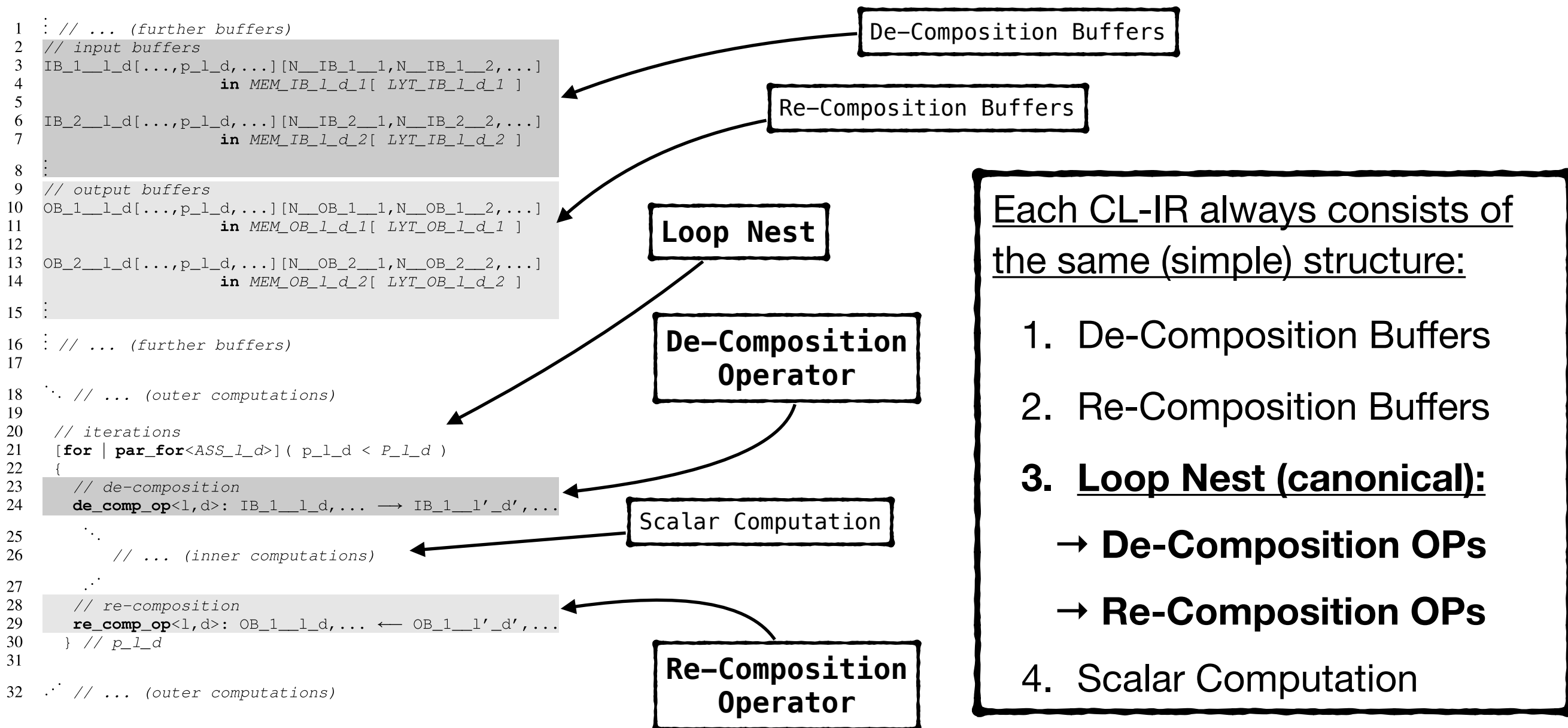
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CL-IR: Code-Level IR

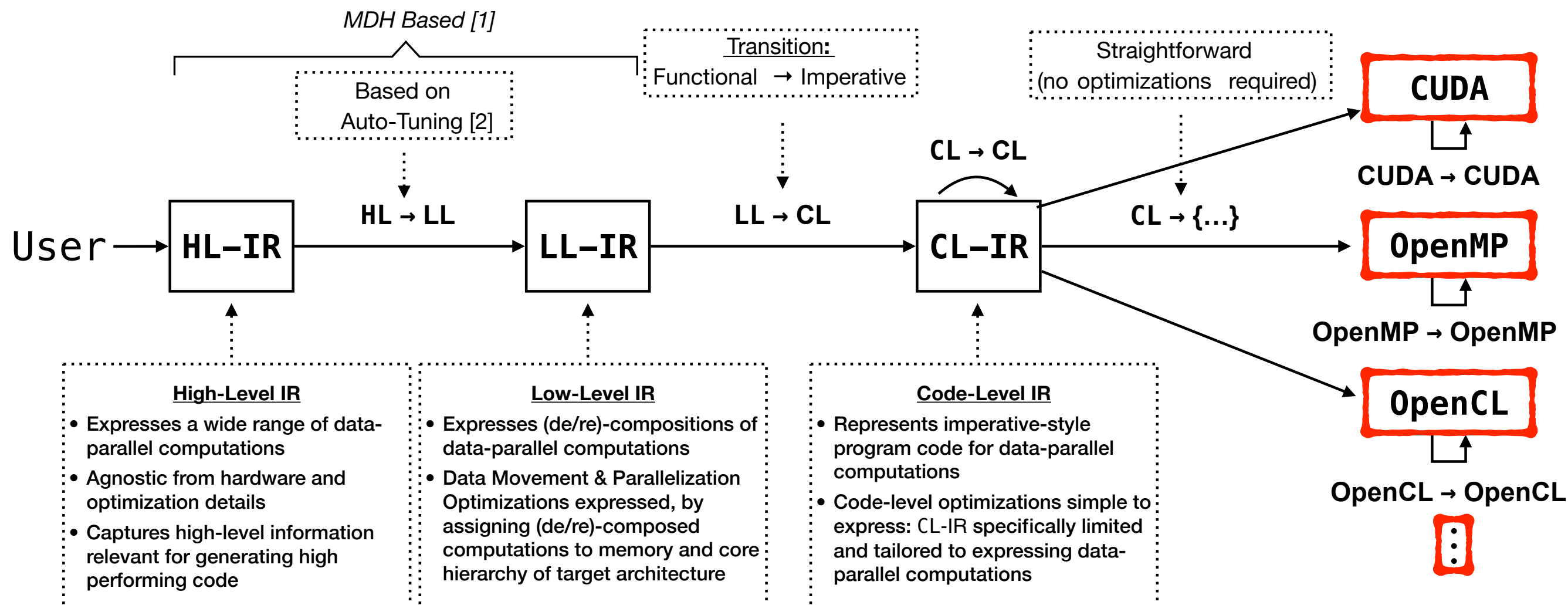
General Structure — (deliberately) limited to expressing ML computations:



Expresses ML computations as
minimalistic, structured, imperative-style code

Overview

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{...}-IR: Backend IRs

We define *minimalistic* IRs for target models, limited to required features:



OpenMP



Currently supported



Triton



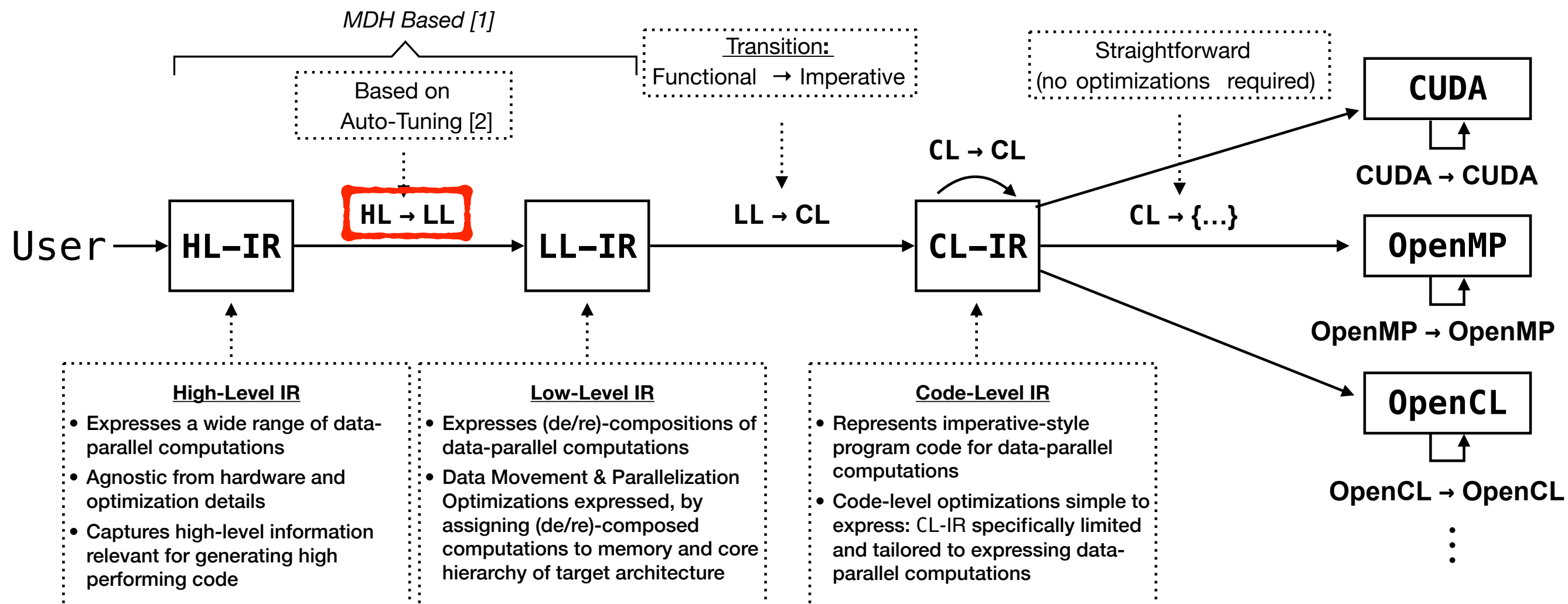
...

Future Targets

Extend CL-IR by **target-specific features** (e.g., tensor core operations)

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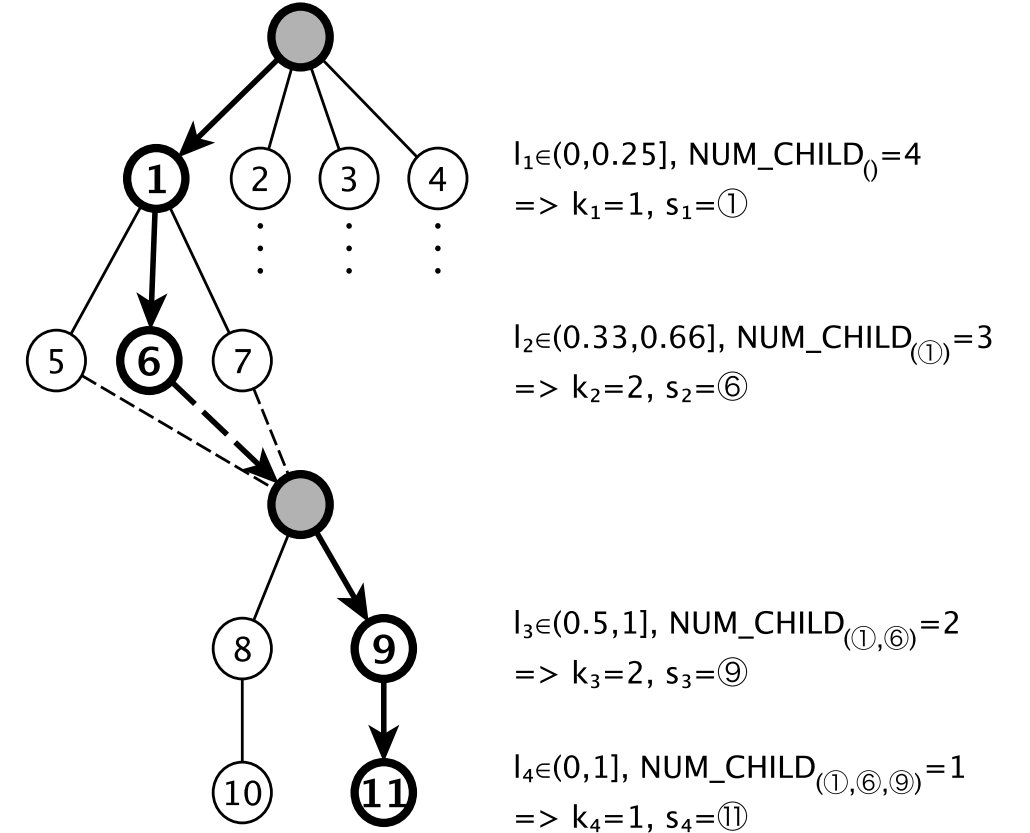
[2] Rasch, Schulze, Steuwer, Gorlatch, “Efficient Auto-Tuning of Parallel Programs with Interdependent Tuning Parameters via Auto-Tuning Framework (ATF)”, TACO’21

Transformation: HL-IR $\rightarrow$ LL-IR

Based on *MDH* [1] and *ATF* [2]:

No.	Name	Range	Description
$\emptyset$	#PRT	$\text{MDH-LVL} \rightarrow \mathbb{N}$	number of parts
D1	$\sigma_{\downarrow\text{-ord}}$	$\text{MDH-LVL} \Leftrightarrow \text{MDH-LVL}$	de-composition order
D2	$\leftrightarrow_{\downarrow\text{-ass}}$	$\text{MDH-LVL} \Leftrightarrow \text{ASM-LVL}$	ASM assignment (de-composition)
D3	$\downarrow\text{-mem}^{<\text{ib}>}$	$\text{MDH-LVL} \rightarrow \text{MR}$	memory regions of input BUFs (ib)
D4	$\sigma_{\downarrow\text{-mem}}^{<\text{ib}>}$	$\text{MDH-LVL} \rightarrow [1, \dots, D_{\text{ib}}^{\text{IB}}]_{\mathcal{S}}$	memory layouts of input BUFs (ib)
S1	$\sigma_f\text{-ord}$	$\text{MDH-LVL} \Leftrightarrow \text{MDH-LVL}$	scalar function order
S2	$\leftrightarrow_f\text{-ass}$	$\text{MDH-LVL} \Leftrightarrow \text{ASM-LVL}$	ASM assignment (scalar function)
S3	$f\downarrow\text{-mem}^{<\text{ib}>}$	MR	memory region of input BUF (ib)
S4	$\sigma_{f\downarrow\text{-mem}}^{<\text{ib}>}$	$[1, \dots, D_{\text{ib}}^{\text{IB}}]_{\mathcal{S}}$	memory layout of input BUF (ib)
S5	$f\uparrow\text{-mem}^{<\text{ob}>}$	MR	memory region of output BUF (ob)
S6	$\sigma_{f\uparrow\text{-mem}}^{<\text{ob}>}$	$[1, \dots, D_{\text{ob}}^{\text{OB}}]_{\mathcal{S}}$	memory layout of output BUF (ob)
R1	$\sigma_{\uparrow\text{-ord}}$	$\text{MDH-LVL} \Leftrightarrow \text{MDH-LVL}$	re-composition order
R2	$\leftrightarrow_{\uparrow\text{-ass}}$	$\text{MDH-LVL} \Leftrightarrow \text{ASM-LVL}$	ASM assignment (re-composition)
R3	$\uparrow\text{-mem}^{<\text{ob}>}$	$\text{MDH-LVL} \rightarrow \text{MR}$	memory regions of output BUFs (ob)
R4	$\sigma_{\uparrow\text{-mem}}^{<\text{ob}>}$	$\text{MDH-LVL} \rightarrow [1, \dots, D_{\text{ob}}^{\text{OB}}]_{\mathcal{S}}$	memory layouts of output BUFs (ob)

MDH-Based Tuning Parameters [1]



ATF-Based Search Space Exploration [2]

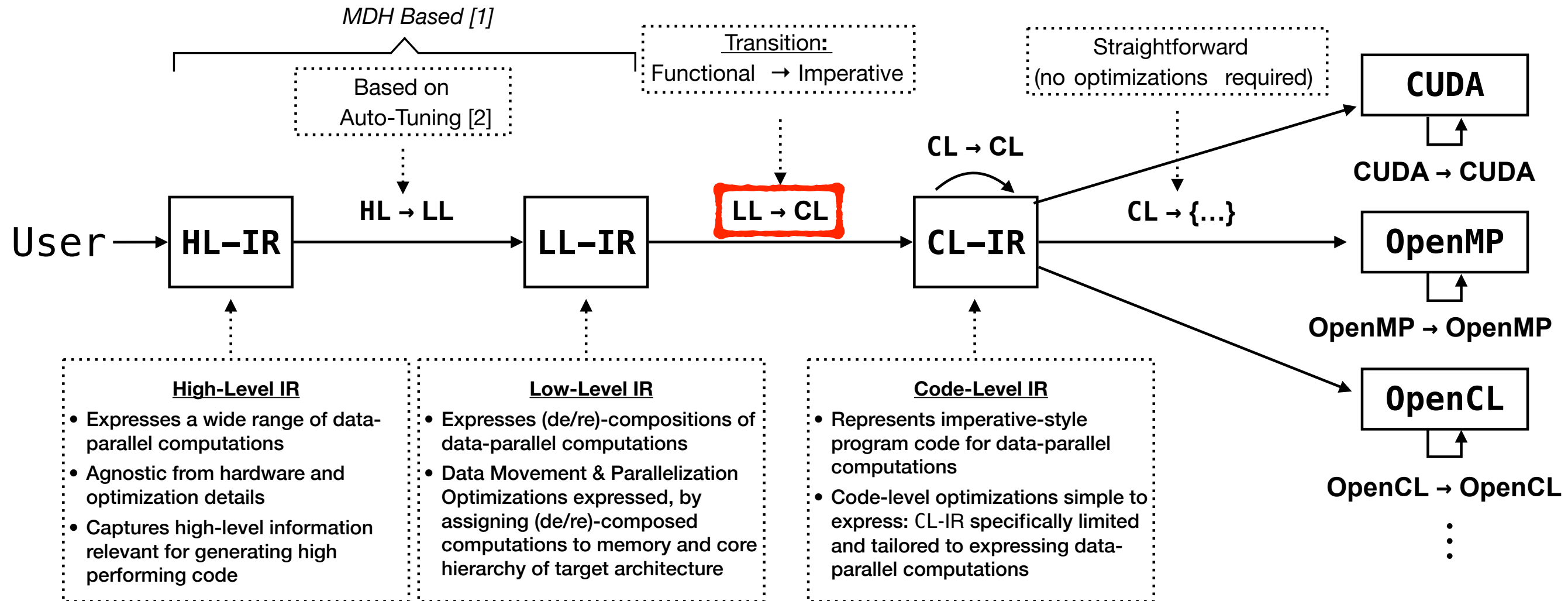
Formally defined transformation driven by performance-critical parameters [1] and auto-tuning [2]

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Transformation: LL-IR $\rightarrow$ CL-IR

Shift from *functional* to *imperative* semantics:

De-Compositions:

- upper part of loops
- copy tiles of input data

Scalar Comp.:

- middle part of loops
- compute scalar function

Re-Compositions:

- lower part of loops
- combine computed intermediate results

```
kernel matvec( w_0_0, M_0_0, v_0_0 )
```

```
{
  ...

```

```
  M_1_2[2][I,K] in HM[1,2]
  v_1_2[2][K]   in HM[1]
  w_1_2[2][I]   in HM[1]

```

```
  ...

```

```
  ...

```

```
  for( p_1_2 < 4 ){
    de_comp_op<1,2>: M_1_2, v_1_2  $\rightarrow$  M_2_1, v_2_1

```

```
    ...

```

```
    w_f = f( (M_f, v_f) )

```

```
    ...

```

```
    re_comp_op<1,2>: w_1_2  $\leftarrow$  w_2_1
  } // p_1_2

```

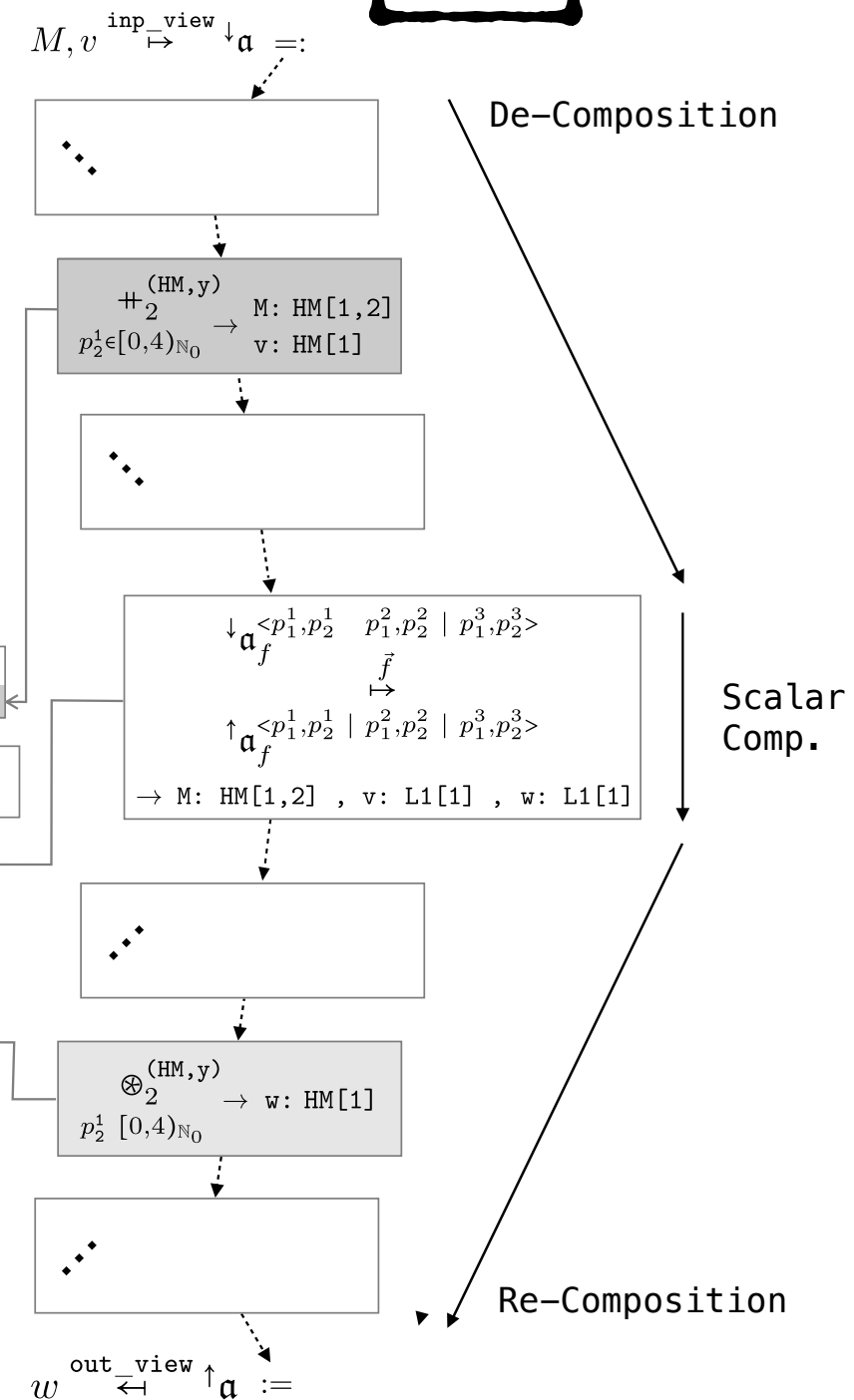
```
    ...

```

```
}
```

CL-IR

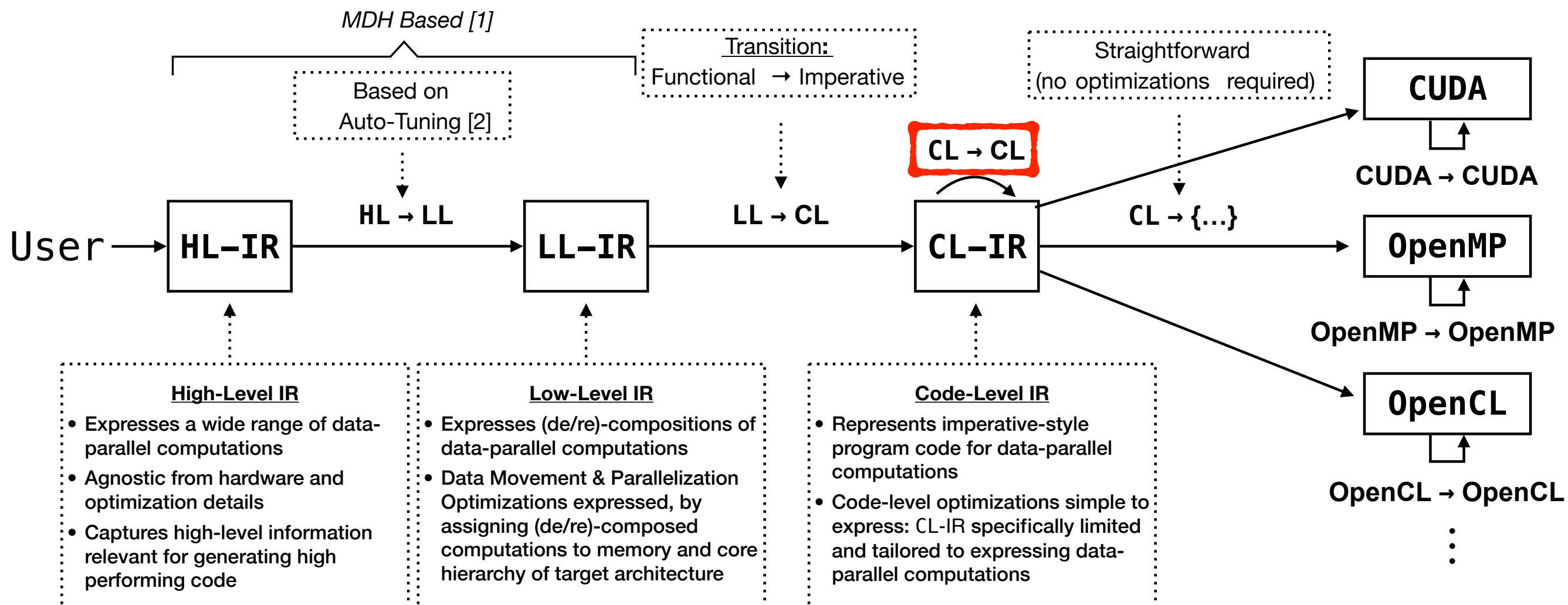
LL-IR



CL-IR instance generated as **sound by construction**
(numerous intermediate buffers, over-approximated buffer sizes, ...)

Overview

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Transformation: CL-IR → CL-IR

Code-Level Optimizations:

Deterministic

Eliminate Functional Overhead

COElimination

Partition Index Elimination

Buffer Size Reduction ...

Standard Code-Level Optimizations

Algebraic Index Simplifications

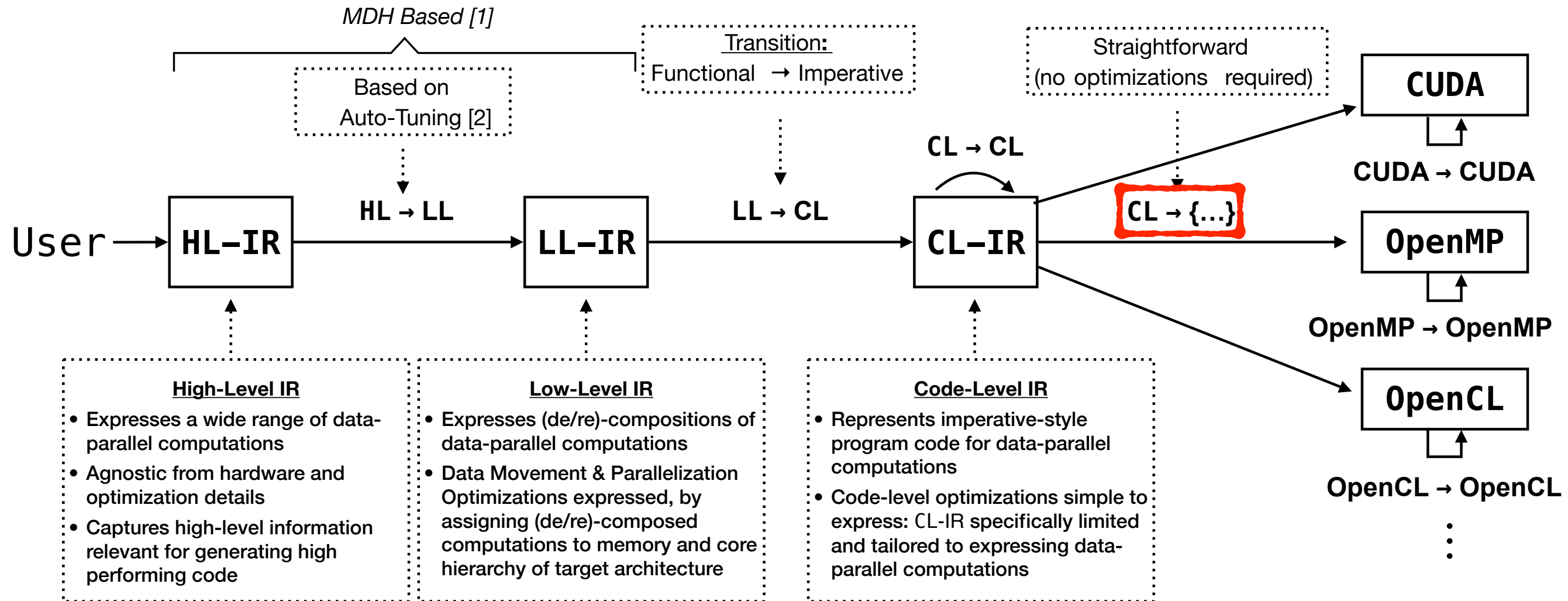
Loop Unrolling

Function Inlining ...

Minimalistic design of CL-IR
simplifies expressing code-level optimizations

Overview

End-to-end ML Code-Generation approach:



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Transformation: CL-IR $\rightarrow$ {...}

Straightforward – all major optimization decisions expressed in earlier steps:

```
// ...  
M_1_1[2][I,K] in SM[1,2]  
v_1_1[2][K]   in RM[1]  
w_1_1[2][I]   in RM[1]  
// ...  
  
for(p_1_2 < 4){  
  par_for<3,1>(p_2_1 < 8){  
    // ...  
  }  
}
```

CL-IR



```
#define T_INP float  
#define T_OUT float  
#define I 1024  
#define K 128  
// ...  
__shared__ T_INP M_1_1[2][I][K];  
           T_INP v_1_1[2][K];  
           T_OUT w_1_1[2][I];  
// ...  
  
for(int p_1_2=0; p_1_2<4; ++p_1_2){  
  int p_2_1 = threadIdx.x;{  
    // ...  
  }  
}
```

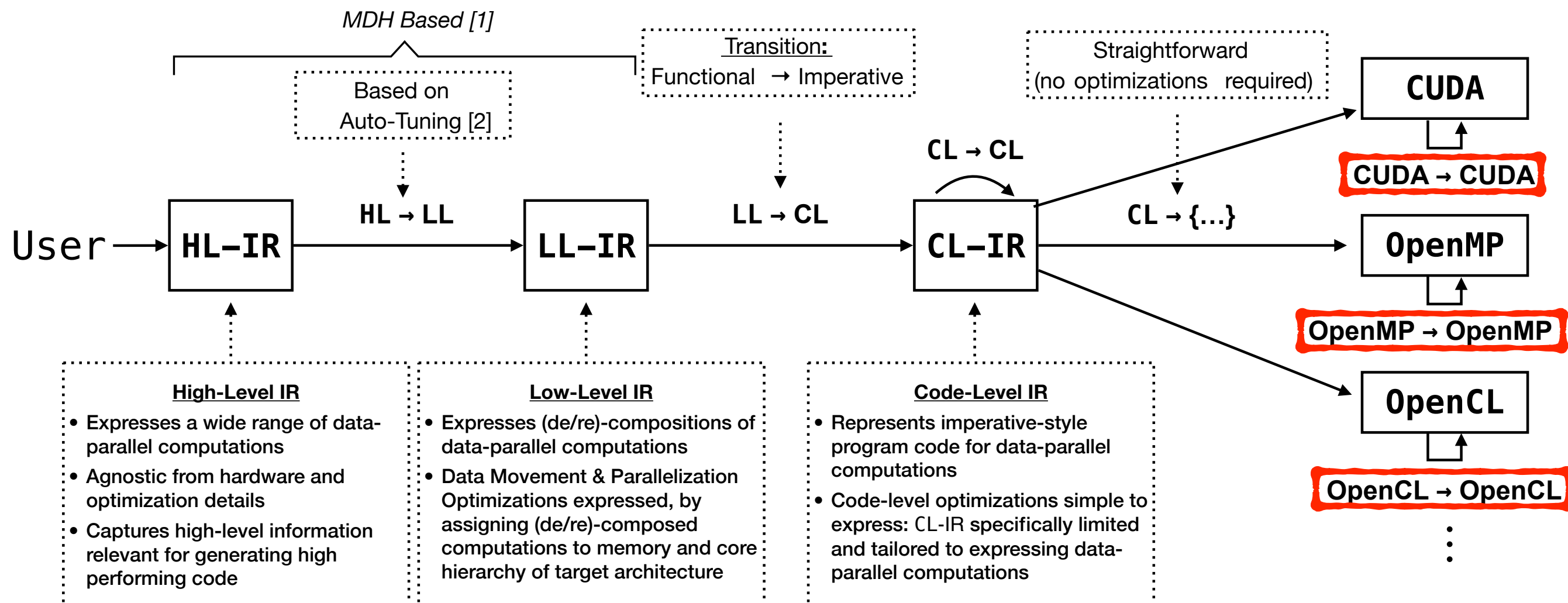


CUDA

Designed to be **straightforward**, allowing
easy extension to new target ML programming models

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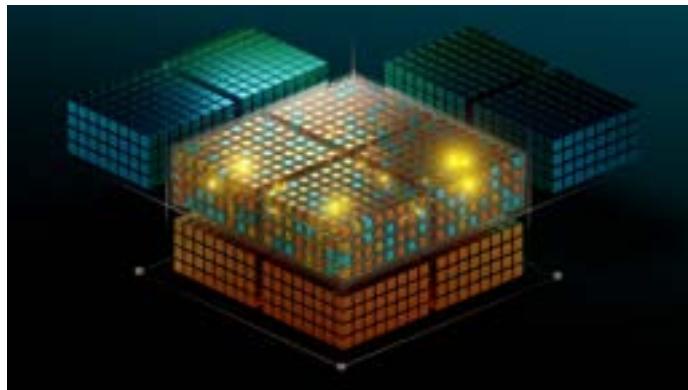
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Transformation: {...} → {...}

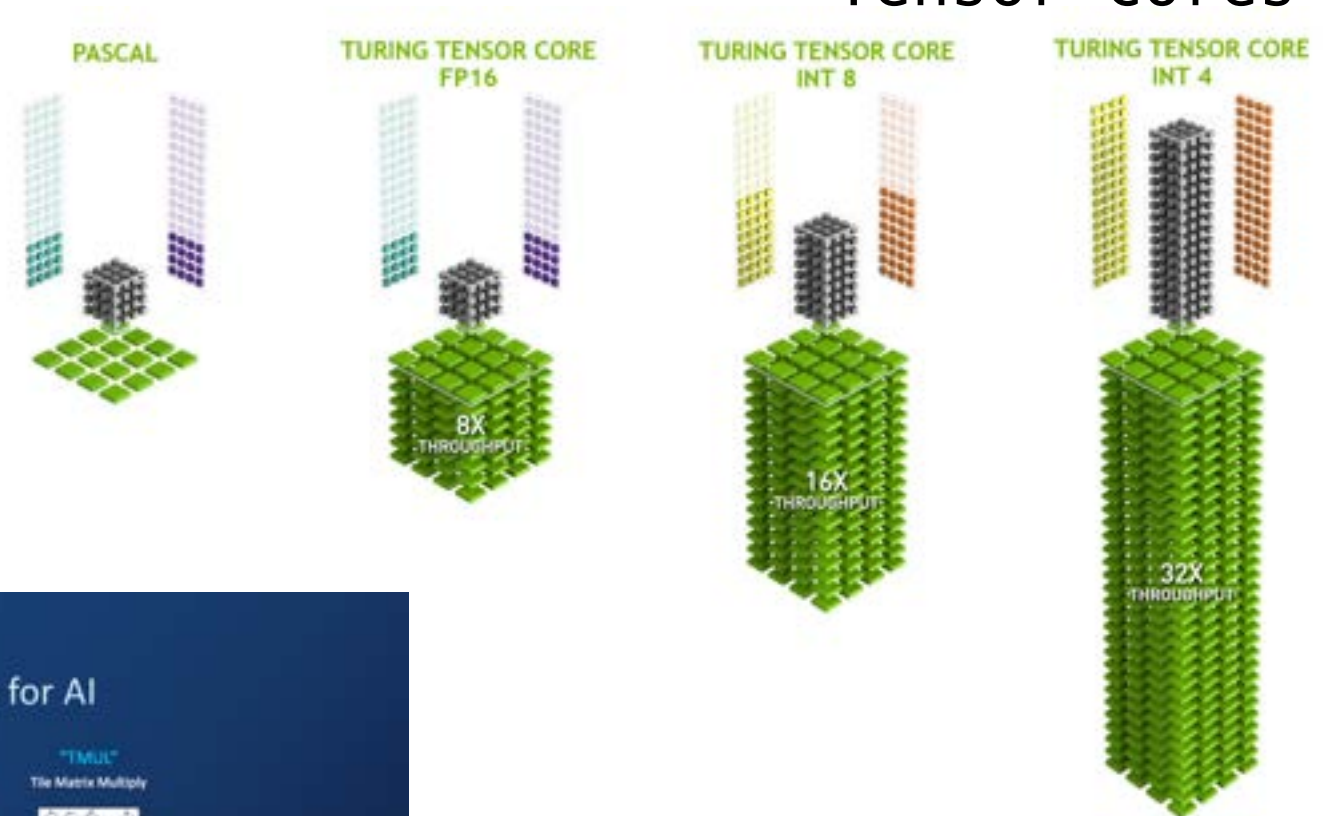


Final, target-specific optimizations:

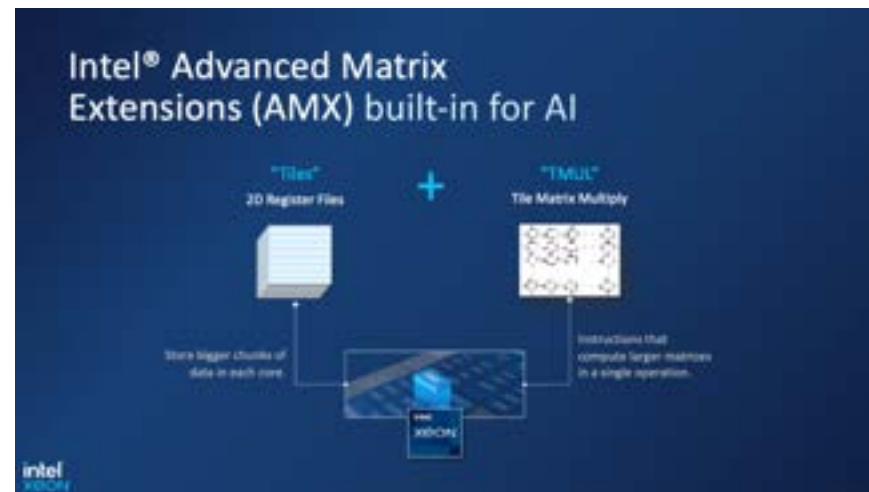
AMD
Matrix Cores



NVIDIA
Tensor Cores



Intel
AMX



Applies **target-specific optimizations** that
exceed the intended generality of the CL-IR abstraction

Experimental Results

Experimental evaluation in terms of **Performance** & *Portability* & *Productivity*:

Competitors:

1. Scheduling Approach:

- Apache TVM [1] (GPU & CPU)

2. Polyhedral Compilers:

- PPCG [2] (GPU)
- Pluto [3] (CPU)

3. Domain-Specific Libraries:

- NVIDIA cuBLAS & cuDNN (GPU)
- Intel oneMKL & oneDNN (CPU)



[1] Chen et al., “TVM: An Automated End-to-End Optimizing Compiler for Deep Learning”, OSDI’18

[2] Verdoolaege et al., “Polyhedral Parallel Code Generation for CUDA”, TACO’13

[3] Bondhugula et al., “PLuTo: A Practical and Fully Automatic Polyhedral Program Optimization System”, PLDI’08

ML Kernels:

1. Linear Algebra Routines:

- Dot Product (Dot)
- Matrix-Vector Multiplication (MatVec)
- Matrix Multiplication (MatMul)
- Matrix Multiplication Transposed (MatMul^T)
- batched Matrix Multiplication (bMatMul)

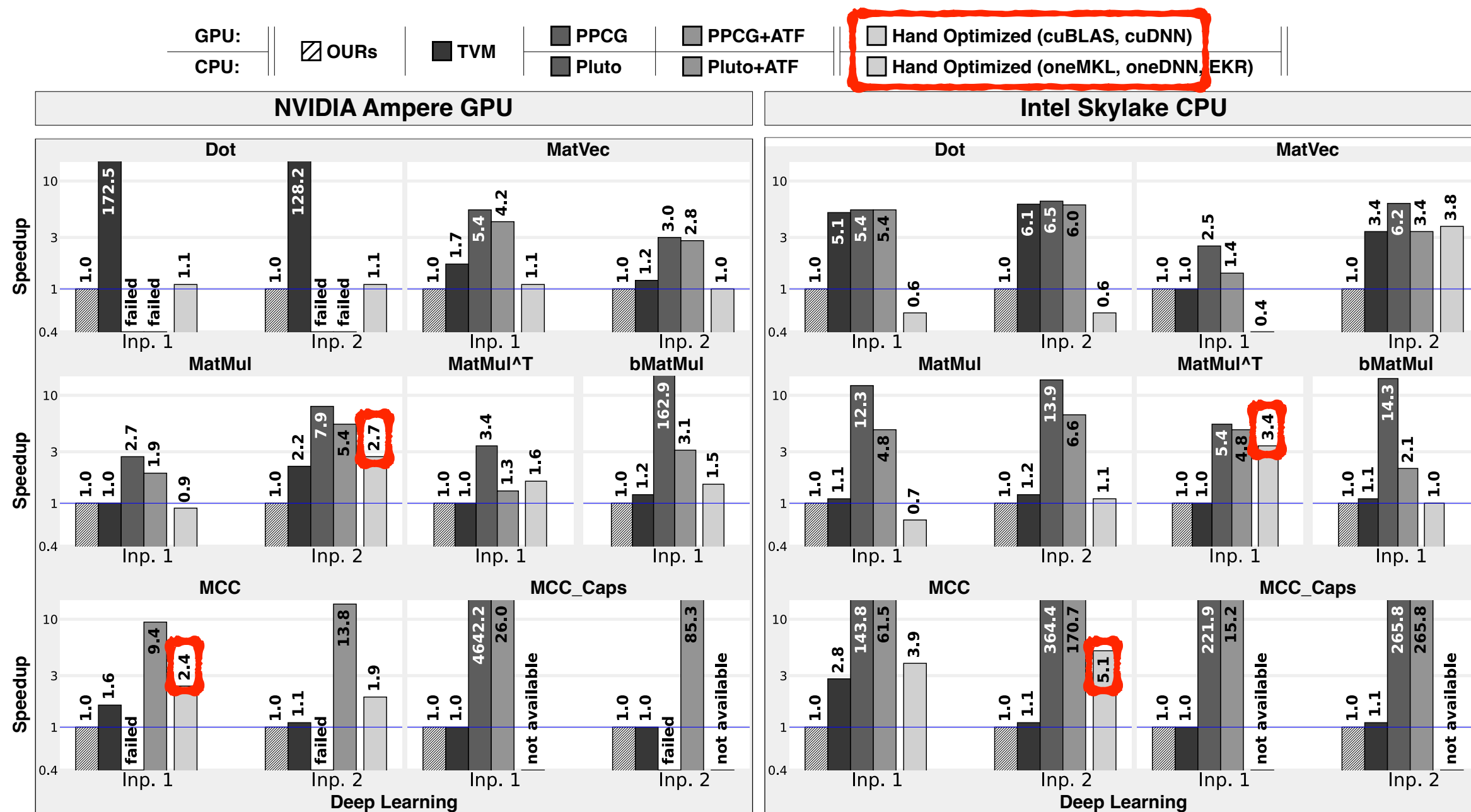
2. Convolutions:

- Multi-Channel Convolution (MCC)
- Capsule-Style Convolution (MCC_Capsule)

Computation	No.	Data Characteristics		
		Sizes		Basic Type
Dot	1	2 ²⁴	2 ²⁴	fp32
	2	10 ⁷	10 ⁷	fp32
MatVec	1	4096x4096	4096	fp32
	2	8192x8192	8192	fp32
MatMul	1	1024x1024	1024x1024	fp32
	2	1x2048	2048x1000	fp32
MatMul ^T	1	64x10	500x64	fp32
bMatMul	1	16x10x64	16x64x500	fp32
MCC	1	1x512x7x7	512x512x3x3	fp32
	2	1x230x230x3	64x7x7x3	fp32
MCC_Caps	1	16x230x230x3x4x4	64x7x7x3x4x4	fp32
	2	1x230x230x3x4x4	67x7x7x3x4x4	fp32

Experimental Results

Performance results for ML kernels on GPU and CPU:



We achieve **high performance** compared to state-of-the-art **machine-** and **hand-optimized approaches**

Summary

We introduce a systematic ML code generation process:

- **Fully automatic**, by separating optimization concerns across abstraction levels
- Designed to be systematically **extensible for new target ML models** (Triton, etc)
- **Formal** foundation, based on algebraic MDH formalism
- **High performance** on **different architectures** (including GPUs and CPUs)

Future Work:

- Show how our design contributes to **aggressive kernel fusion** optimization
- Exploit **ML-specific hardware** extensions
- **Assembly**-level targets (PTX, ...)
- **Sparse** Computations
- ...

Questions?



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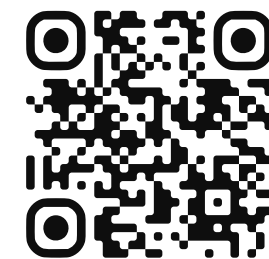
<https://mdh-lang.org>

**Code
Generation**



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**Code
Optimization**



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