Universität Münster

roofline

MDH+ATF: Code *Generation & Optimization* for Deep-Learning Computations

Ari Rasch, Richard Schulze
University of Münster, Germany

Who are we?

Main Projects:

https://mdh-lang.org

MDH

Code Generation

https://arirasch.net
a.rasch@uni-muenster.de

Ari Rasch

https://atf-tuner.org

ATF

Code Optimization

http://hca-project.org

HCA

Code Execution

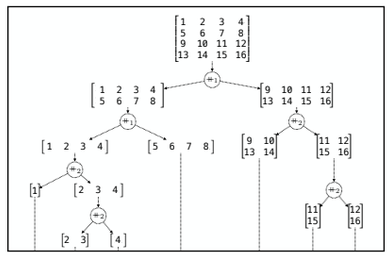
Hunloh Brothers (Lars & Jens)

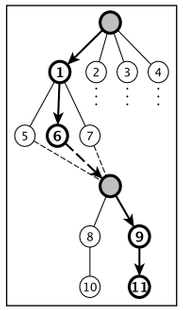
https://richardschulze.net
r.schulze@uni-muenster.de

Richard Schulze

Who are we?

Responsibilities/Competencies:





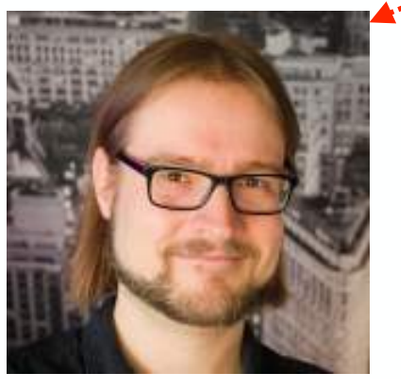
```
import numpy as np

from pyatf import TP, Interval, Tuner
from pyatf.cost_functions import cuda
from pyatf.search_techniques import AUCBandit
from pyatf.abort_conditions import Evaluations

# input size
N = 1000

# Step 1: Generate the Search Space
WPT = TP('WPT', Interval( 1, N ), lambda WPT: N % WPT == 0          )
LS  = TP('LS',  Interval( 1, N ), lambda WPT, LS: (N / WPT) % LS == 0)

# Step 2: Implement a Cost Function
saxpy_kernel = cuda.Kernel( cuda.source(saxpy_kernel_as_string), "
```

Lead Developer

MDH ATF HCA

Lead Engineer

Engineers (much MLIR)

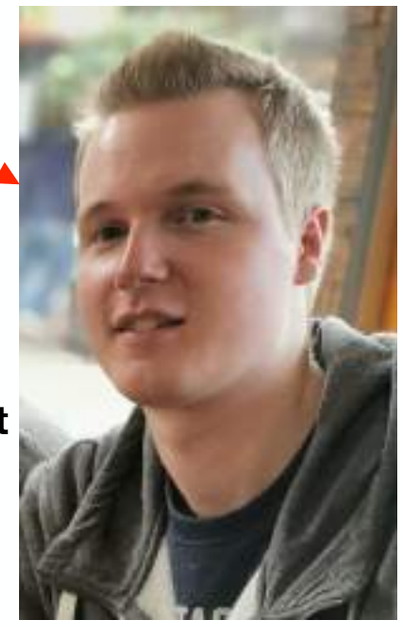


Hunloh Brothers (Lars & Jens)

https://arirasch.net
a.rasch@uni-muenster.de

Ari Rasch

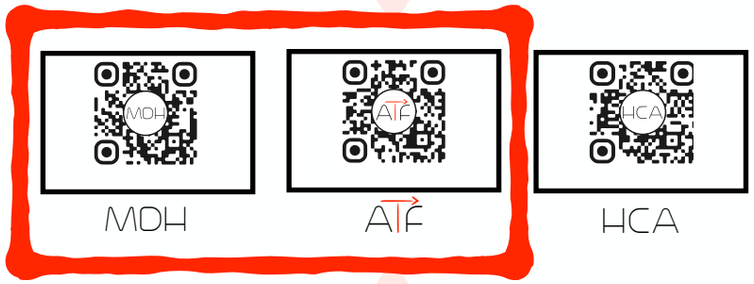
https://richardschulze.net
r.schulze@uni-muenster.de



Richard Schulze

Who are we?

Responsibilities/Competencies:



https://arirasch.net
a.rasch@uni-muenster.de

Ari Rasch

Focus Today

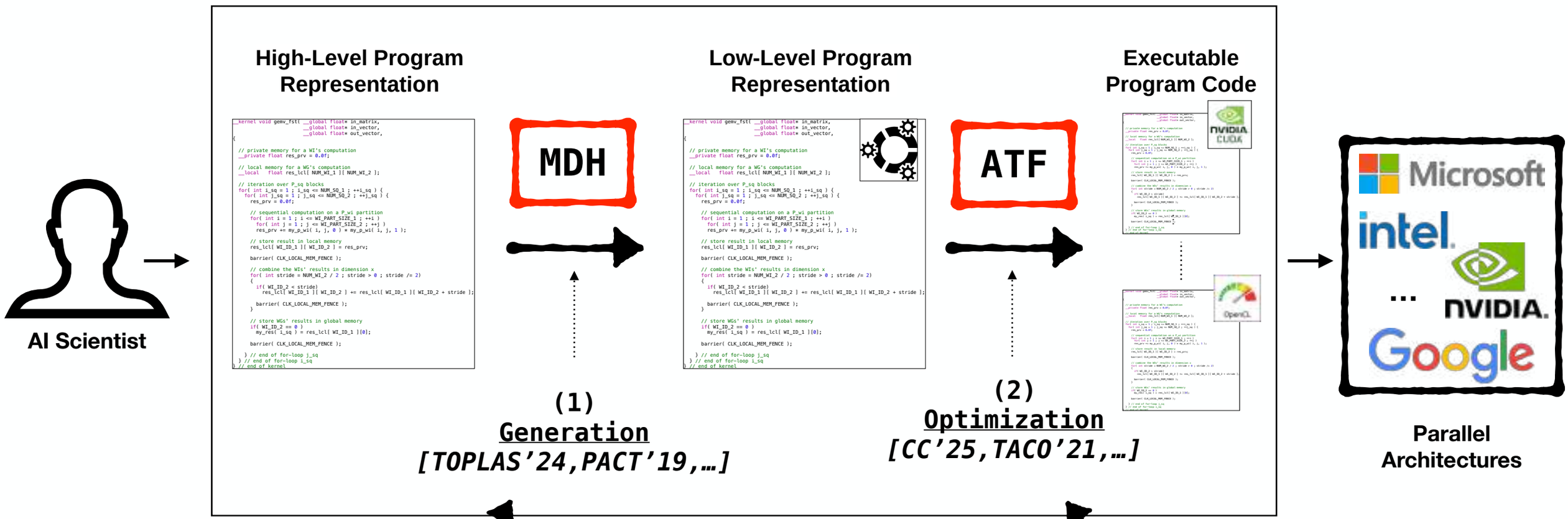
https://richardschulze.net
r.schulze@uni-muenster.de

Richard Schulze

Hunloh Brothers (Lars & Jens)

Goal of MDH+ATF

An approach to ***Generating (MDH)*** & ***Optimizing (ATF)*** code for **DL computations**:

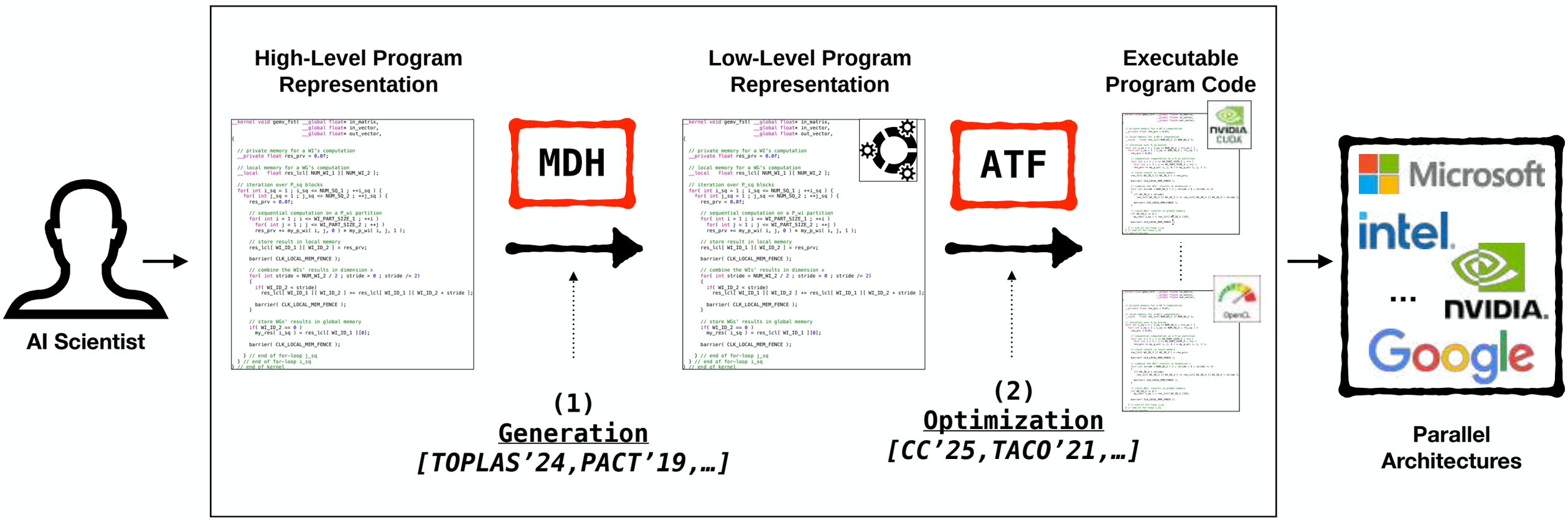


1. Part: How to generate automatically optimizable (auto-tunable) DL code?

2. Part: How to optimize (auto-tune) DL code?

Goal of MDH+ATF

An approach to ***Generating (MDH)*** & ***Optimizing (ATF)*** code for **DL computations**[1]:



The ultimate goal of **MDH+ATF** is to simultaneously achieve
Performance & **Portability** & **Productivity**
for **DL computations**[1]

[1] MDH not limited to DL: targets arbitrary kinds of data-parallel computations

Agenda

1. MDH

   - MDH (formal) *Program Representation* & *Experimental Results*
   - MDH *Code Generation* & *Interfaces*:
     - MDH *Python*-Based Interface
     - MDH *MLIR*-Based Interface & Comparison to *Linalg*
   - MDH Future Work

2. ATF

   - Motivation & Usage (briefly)

3. roofline — Possible Collaboration(s)

https://mdh-lang.org

MDH

Code Generation

https://atf-tuner.org

ATF

Code Optimization

Code Generation via MDH

Overview Getting Started Code Examples Publications Citations Contact

MDH

Multi-Dimensional Homomorphisms (MDH)

An Algebraic Approach Toward Performance & Portability & Productivity for Data-Parallel Computations

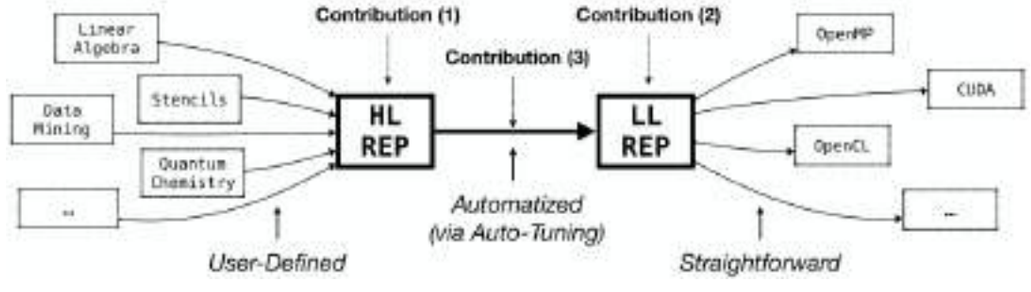
Overview

The approach of Multi-Dimensional Homomorphisms (MDH) is an algebraic formalism for systematically reasoning about *de-composition* and *re-composition* strategies of data-parallel computations (such as linear algebra routines and stencil computations) for the memory and core hierarchies of state-of-the-art parallel architectures (GPUs, multi-core CPU, multi-device and multi-node systems, etc).

The MDH approach (formally) introduces:

1. *High-Level Program Representation (Contribution 1)* that enables the user conveniently implementing data-parallel computations, agnostic from hardware and optimization details;
2. *Low-Level Program Representation (Contribution 2)* that expresses device- and data-optimized de- and re-composition strategies of computations;
3. *Lowering Process (Contribution 3)* that fully automatically lowers a data-parallel computation expressed in its high-level program representation to an optimized instance in its low-level representation, based on concepts from automatic performance optimization (a.k.a. *auto-tuning*), using the Auto-Tuning Framework (ATF).

The MDH's low-level representation is designed such that Code Generation from it (e.g., in OpenMP for CPUs, CUDA for NVIDIA GPUs, or OpenCL for multiple kinds of architectures) becomes straightforward.



Our Experiments report encouraging results on GPUs and CPUs for MDH as compared to state-of-practice approaches, including NVIDIA cuBLAS/cuDNN and Intel oneMKL/oneDNN which are hand-optimized libraries provided by vendors.

https://mdh-lang.org

ACM TOPLAS 2024

(De/Re)-Composition of Data-Parallel Computations via Multi-Dimensional Homomorphisms

ARI RASCH, University of Muenster, Germany

Data-parallel computations, such as linear algebra routines and stencil computations, constitute one of the most relevant classes in parallel computing, e.g., due to their importance for deep learning. Efficiently de-composing such computations for the memory and core hierarchies of modern architectures and re-composing the computed intermediate results back to the final result—we say *(de/re)-composition* for short—is key to achieve high performance for these computations on, e.g., GPU and CPU. Current high-level approaches to generating data-parallel code are often restricted to a particular subclass of data-parallel computations and architectures (e.g., only linear algebra routines on only GPU or only stencil computations), and/or the approaches rely on a user-guided optimization process for a well-performing (de/re)-composition of computations, which is complex and error prone for the user.

We formally introduce a systematic (de/re)-composition approach, based on the algebraic formalism of Multi-Dimensional Homomorphisms (MDHs). Our approach is designed as general enough to be applicable to a wide range of data-parallel computations and for various kinds of target parallel architectures. To efficiently target the deep and complex memory and core hierarchies of contemporary architectures, we exploit our introduced (de/re)-composition approach for a correct-by-construction, parametrized cache blocking, and parallelization strategy. We show that our approach is powerful enough to express, in the same formalism, the (de/re)-composition strategies of different classes of state-of-the-art approaches (scheduling-based, polyhedral, etc.), and we demonstrate that the parameters of our strategies enable systematically generating code that can be fully automatically optimized (auto-tuned) for the particular target architecture and characteristics of the input and output data (e.g., their sizes and memory layouts). Particularly, our experiments confirm that via auto-tuning, we achieve higher performance than state-of-the-art approaches, including hand-optimized solutions provided by vendors (such as NVIDIA cuBLAS/cuDNN and Intel oneMKL/oneDNN), on real-world datasets and for a variety of data-parallel computations, including linear algebra routines, stencil and quantum chemistry computations, data mining algorithms, and computations that recently gained high attention due to their relevance for deep learning.

CCS Concepts: • **Computing methodologies** → **Parallel computing methodologies**; *Machine learning*; • **Theory of computation** → **Program semantics**; • **Software and its engineering** → **Compilers**;

Additional Key Words and Phrases: Code generation, data parallelism, auto-tuning, GPU, CPU, OpenMP, CUDA, OpenCL, linear algebra, stencils computation, quantum chemistry, data mining, deep learning

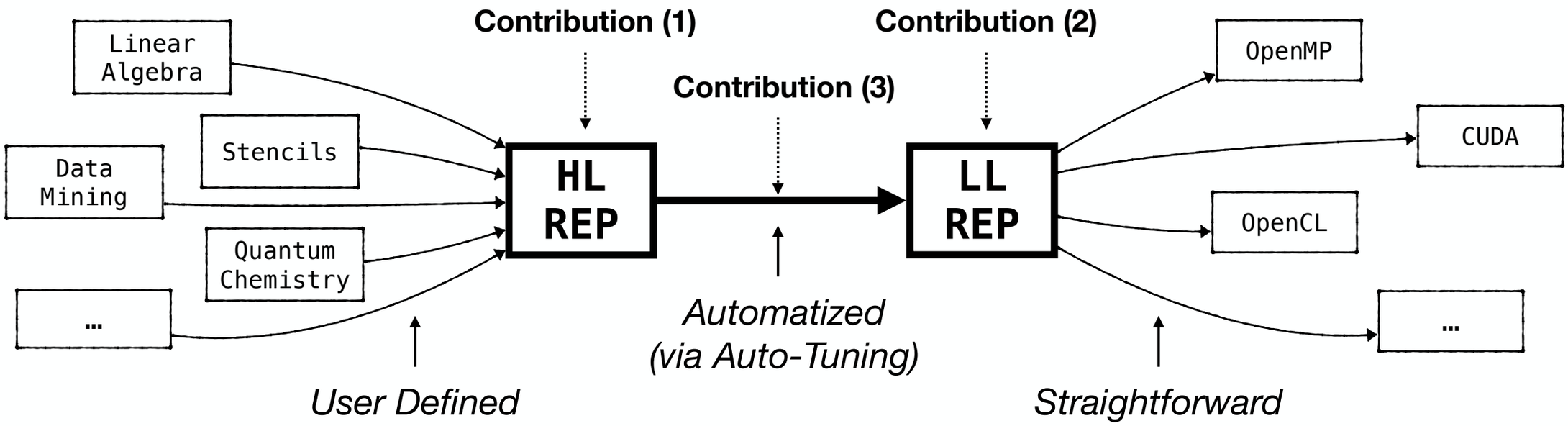
A full version of this article is provided by Rasch [2024], which presents our novel concepts with all of their formal details. In contrast to the full version, this article relies on a simplified formal foundation for better illustration and easier understanding. We often refer the interested reader to Rasch [2024] for formal details that should not be required for understanding the basic ideas and concepts of our approach.

This work was funded by the Deutsche Forschungsgemeinschaft (DFG, German Research Foundation)—project PPP-DL (470527619).

Author's Contact Information: Ari Rasch (Corresponding author), University of Muenster, Muenster, Germany; e-mail: a.rasch@uni-muenster.de.

Goal of **MDH**

MDH is a (formal) framework for *expressing* & *optimizing* data-parallel computations:



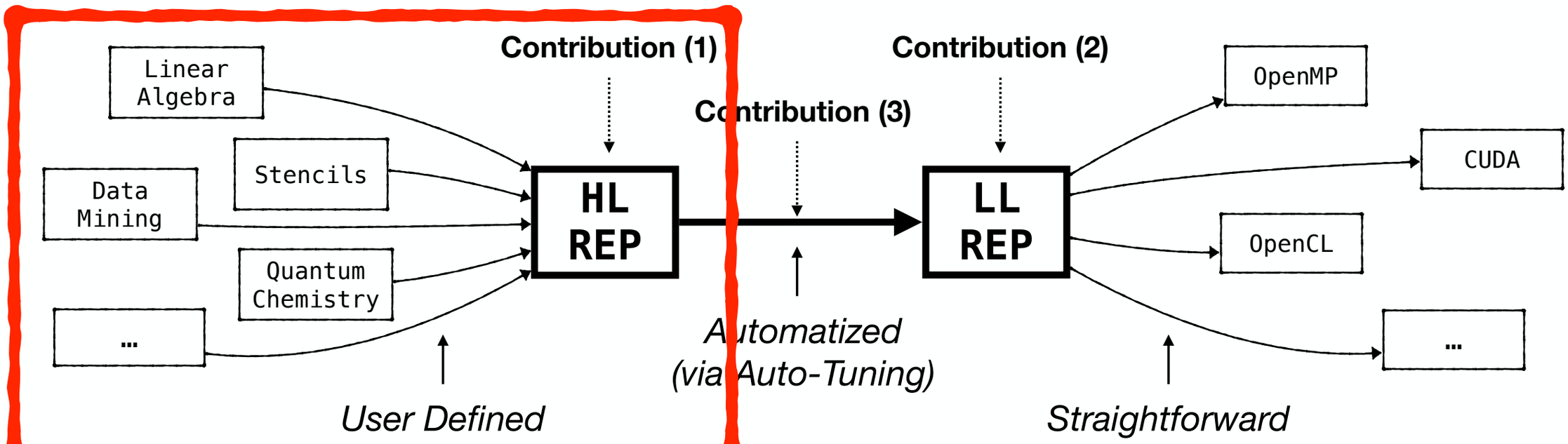
1. ***Contribution 1 (HL-REP):*** defines *data parallelism*, based on *common algebraic properties of computations* & introduces *higher-order functions* for expressing these computations, agnostic from hardware and optimization details while still capturing high-level information relevant for generating high-performing code

2. ***Contribution 2 (LL-REP):*** allows *expressing and reasoning about optimizations* for the memory and core hierarchies of contemporary parallel architectures & generalizes these optimizations to apply to arbitrary combinations of data-parallel computations and architectures

3. ***Contribution 3 (→):*** introduces a *structured optimization process* — for arbitrary combinations of data-parallel computations and parallel architectures — that allows *fully automatic* optimization (auto-tuning)

Goal of **MDH**

MDH is a (formal) framework for *expressing* & *optimizing* data-parallel computations:



1. ***Contribution 1 (HL-REP):*** defines *data parallelism*, based on *common algebraic properties of computations* & introduces *higher-order functions* for expressing these computations, agnostic from hardware and optimization details while still capturing high-level information relevant for generating high-performing code

2. ***Contribution 2 (LL-REP):*** allows *expressing and reasoning about optimizations* for the memory and core hierarchies of contemporary parallel architectures & generalizes these optimizations to apply to arbitrary combinations of data-parallel co

Focus Today

3. ***Contribution 3 (→):*** introduces for arbitrary combinations of data-parallel computations and parallel architectures — that allows *fully automatic* optimization (auto-tuning)

MDH: High-Level Representation

Goals:

1. **Uniform**:
   should be able to express any kind of data-parallel computation, without relying on domain-specific building blocks, extensions, etc.

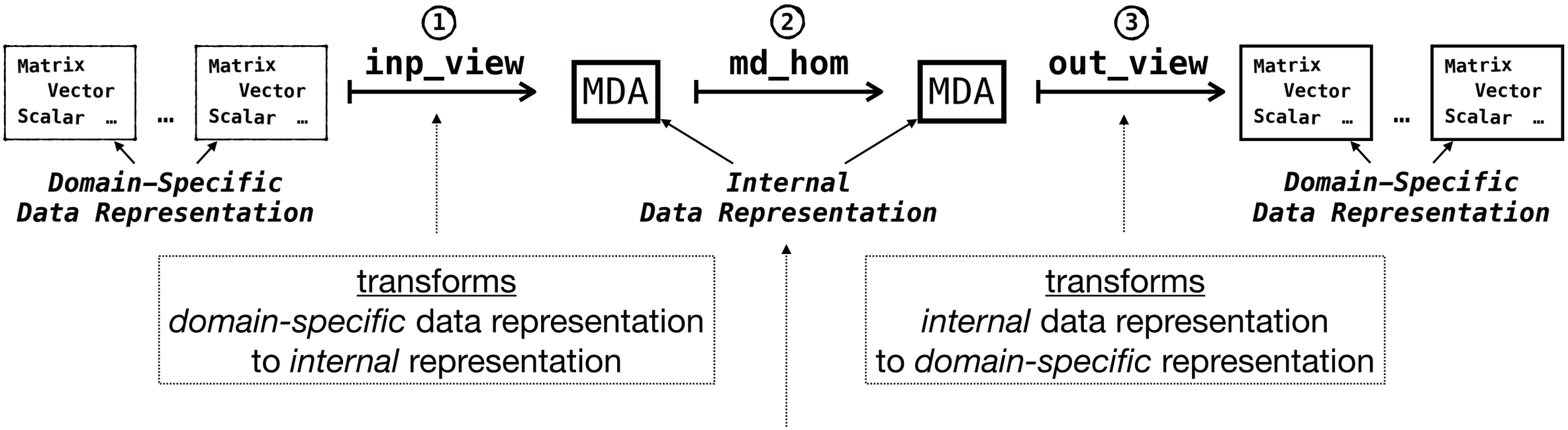
2. **Minimalistic**:
   should rely on less building blocks to keep language small and simple

3. **Structured**:
   avoiding compositions and nestings of building blocks as much as possible, thereby further contributing to the usability and simplicity of our language

```
MatVec^<T∈TYPE|I,K∈ℕ>  :=  out_view<T>( w:(i,k)↦(i) ) ∘
                                md_hom<I,K>( *, (+,+) ) ∘
                                    inp_view<T,T>( M:(i,k)↦(i,k) , v:(i,k)↦(k) )
```

***MDH High-Level Representation of* MatVec**
(*discussed later*)

MDH: High-Level Representation



Our high-level representation (formally) defines DL computations[1],
and it expresses these computations using exactly
three straightforwardly composed higher-order functions only

[1] MDH not limited to DL: targets arbitrary kinds of data-parallel computations

MDH: High-Level Representation

Example: `MatVec` expressed in MDH

```
MatVec^<T∈TYPE|I,K∈ℕ> := out_view<T>( w:(i,k)↦(i) ) ∘
                          md_hom<I,K>( *, (+,+) ) ∘
                          inp_view<T,T>( M:(i,k)↦(i,k) , v:(i,k)↦(k) )
```

MDH

***MDH High-Level Representation of* MatVec**

What is happening here:

- `inp_view` captures the accesses to input data
- `md_hom` expresses the data-parallel computation
- `out_view` captures the accesses to output data

```
void MatVec( T[] M, T[] v, T[] w )
{
  for( int i=0 ; i < I ; ++i )
    for( int k=0 ; k < K ; ++k )
      w[i] += M[i][k] * v[k];
}
```

MatVec in C++

[1]We can generate such MDH expressions also automatically from straightforward (annotated) code in Python, C, …

MDH: High-Level Representation

md_hom	f	$\circledast_1, \ldots, \circledast_D$
MatMul<F,F>	$*$	$+\!\!+, +\!\!+, +$
MatMul<F,T>	$*$	$+\!\!+, +\!\!+, +$
MatMul<T,F>	$*$	$+\!\!+, +\!\!+, +$
MatMul<T,T>	$*$	$+\!\!+, +\!\!+, +$
BatchMatMul<F,F>	$*$	$+\!\!+, \ldots, +\!\!+, +$
⋮	⋮	⋮
BiasAddGrad<NHWC>	id	$+, +, +, +\!\!+$
BiasAddGrad<NCHW>	id	$+, +\!\!+, +, +$
CheckNumerics	$(x) \mapsto (x == \text{NaN})$	$\vee, \ldots, \vee$
Sum<0><F>	id	$+, +\!\!+, +\!\!+, \ldots, +\!\!+$
Sum<0><T>	id	$+, +\!\!+, +\!\!+, \ldots, +\!\!+$
Sum<1><F>	id	$+\!\!+, +, +\!\!+, \ldots, +\!\!+$
Sum<0,1><F>	id	$+, +, +\!\!+, \ldots, +\!\!+$
⋮	⋮	⋮
Prod<0><F>	id	$*, +\!\!+, +\!\!+, \ldots, +\!\!+$
⋮	⋮	⋮
All<0><F>	id	$\&\&, +\!\!+, +\!\!+, \ldots, +\!\!+$
⋮	⋮	⋮

Linear Algebra, Reductions, … (Non-Endomorphic Operators)
— Computation Specification —

Views	inp_view I_1	inp_view I_2	out_view O
MatMul<F,F>	$(i,j,k) \mapsto (i,k)$	$(i,j,k) \mapsto (k,j)$	$(i,j,k) \mapsto (i,j)$
MatMul<F,T>	$(i,j,k) \mapsto (i,k)$	$(i,j,k) \mapsto (j,k)$	$(i,j,k) \mapsto (i,j)$
MatMul<T,F>	$(i,j,k) \mapsto (k,i)$	$(i,j,k) \mapsto (k,j)$	$(i,j,k) \mapsto (i,j)$
MatMul<T,T>	$(i,j,k) \mapsto (k,i)$	$(i,j,k) \mapsto (j,k)$	$(i,j,k) \mapsto (i,j)$
BatchMatMul<F,F>	$(b_1,\ldots,i,j,k) \mapsto (b_1,\ldots,i,k)$	$(b_1,\ldots,i,j,k) \mapsto (b_1,\ldots,k,j)$	$(b_1,\ldots,i,j,k) \mapsto (b_1,\ldots,i,j)$
⋮	⋮	⋮	⋮
BiasAddGrad<NHWC>	$(n,h,w,c) \mapsto (n,h,w,c)$	/	$(n,h,w,c) \mapsto (n,h,w)$
BiasAddGrad<NCHW>	$(n,c,h,w) \mapsto (n,c,h,w)$	/	$(n,c,h,w) \mapsto (n,h,w)$
CheckNumerics	$(i_1,\ldots,i_D) \mapsto (i_1,\ldots,i_D)$	/	$(i_1,\ldots,i_D) \mapsto ()$
Sum<0><F>	$(i_1,\ldots,i_D) \mapsto (i_1,\ldots,i_D)$	/	$(i_1,\ldots,i_D) \mapsto (i_2,\ldots,i_D)$
Sum<0><T>	$(i_1,\ldots,i_D) \mapsto (i_1,\ldots,i_D)$	/	$(i_1,\ldots,i_D) \mapsto (0,i_2,\ldots,i_D)$
Sum<1><F>	$(i_1,\ldots,i_D) \mapsto (i_1,\ldots,i_D)$	/	$(i_1,\ldots,i_D) \mapsto (i_1,i_3,\ldots,i_D)$
Sum<0,1><F>	$(i_1,\ldots,i_D) \mapsto (i_1,\ldots,i_D)$	/	$(i_1,\ldots,i_D) \mapsto (i_3,\ldots,i_D)$
⋮	⋮	⋮	⋮
Prod<0><F>	$(i_1,\ldots,i_D) \mapsto (i_1,\ldots,i_D)$	/	$(i_1,\ldots,i_D) \mapsto (i_2,\ldots,i_D)$
⋮	⋮	⋮	⋮
All<0><F>	$(i_1,\ldots,i_D) \mapsto (i_1,\ldots,i_D)$	/	$(i_1,\ldots,i_D) \mapsto (i_2,\ldots,i_D)$
⋮	⋮	⋮	⋮

Linear Algebra, Reductions, … (Non-Endomorphic Operators)
— Data Specification —

md_hom	f	$\circledast_1, \ldots, \circledast_D$
Fill	id	$+\!\!+, \ldots, +\!\!+$
ExpandDims<0>	id	$+\!\!+, \ldots, +\!\!+$
ExpandDims<1>	id	$+\!\!+, \ldots, +\!\!+$
ExpandDims<0,1>	id	$+\!\!+, \ldots, +\!\!+$
⋮	⋮	⋮
Transpose<σ>	id	$+\!\!+, \ldots, +\!\!+$
Exp	exp	$+\!\!+, \ldots, +\!\!+$
Mul	$*$	$+\!\!+, \ldots, +\!\!+$
BiasAdd<NHWC>	$+$	$+\!\!+, +\!\!+, +\!\!+, +\!\!+$
BiasAdd<NCHW>	$+$	$+\!\!+, +\!\!+, +\!\!+, +\!\!+$
Range	$(s,d,i) \mapsto (s + d * i)$	$+\!\!+$

Point-Wise, Re-Shaping, … (Endomorphic Operators)
— Computation Specification —

Views	inp_view I_1	inp_view I_2	out_view O
Fill	$(i_1,\ldots,i_D) \mapsto ()$	/	$(i_1,\ldots,i_D) \mapsto (i_1,\ldots,i_D)$
ExpandDims<0>	$(i_1,\ldots,i_D) \mapsto (i_1,\ldots,i_D)$	/	$(i_1,\ldots,i_D) \mapsto (0,i_1,i_2,\ldots,i_D)$
ExpandDims<0>	$(i_1,\ldots,i_D) \mapsto (i_1,\ldots,i_D)$	/	$(i_1,\ldots,i_D) \mapsto (i_1,0,i_2,\ldots,i_D)$
ExpandDims<0,1>	$(i_1,\ldots,i_D) \mapsto (i_1,\ldots,i_D)$	/	$(i_1,\ldots,i_D) \mapsto (0,0,i_1,\ldots,i_D)$
⋮	⋮	⋮	⋮
Transpose<σ>	$(i_1,\ldots,i_D) \mapsto (\sigma(i_1),\ldots,\sigma(i_D))$	/	$(i_1,\ldots,i_D) \mapsto (i_1,\ldots,i_D)$
Exp	$(i_1,\ldots,i_D) \mapsto (i_1,\ldots,i_D)$	/	$(i_1,\ldots,i_D) \mapsto (i_1,\ldots,i_D)$
Mul	$(i_1,\ldots,i_D) \mapsto (i_1,\ldots,i_D)$ $(i_1,\ldots,i_D) \mapsto (i_1,\ldots,i_D)$	$(i_1,\ldots,i_D) \mapsto (i_1,\ldots,i_D)$ $(i_1,\ldots,i_D) \mapsto (i_1,\ldots,i_{k-1},i_{k+1},\ldots,i_D)$	$(i_1,\ldots,i_D) \mapsto (i_1,\ldots,i_D)$ $(i_1,\ldots,i_D) \mapsto (i_1,\ldots,i_D)$
⋮	⋮	⋮	⋮
BiasAdd<NHWC>	$(n,h,w,c) \mapsto (n,h,w,c)$	$(n,h,w,c) \mapsto (c)$	$(n,h,w,c) \mapsto (n,h,w,c)$
BiasAdd<NCHW>	$(n,c,h,w) \mapsto (n,c,h,w)$	$(n,c,h,w) \mapsto (c)$	$(n,c,h,w) \mapsto (n,c,h,w)$
Range	$(i) \mapsto ()$	$(i) \mapsto ()$	$(i) \mapsto (i)$

Point-Wise, Re-Shaping, … (Endomorphic Operators)
— Data Specification —

Important DL computations can be (naturally & uniformly) expressed in MDH

MDH: Formalism

That was a very quick, informal dive!

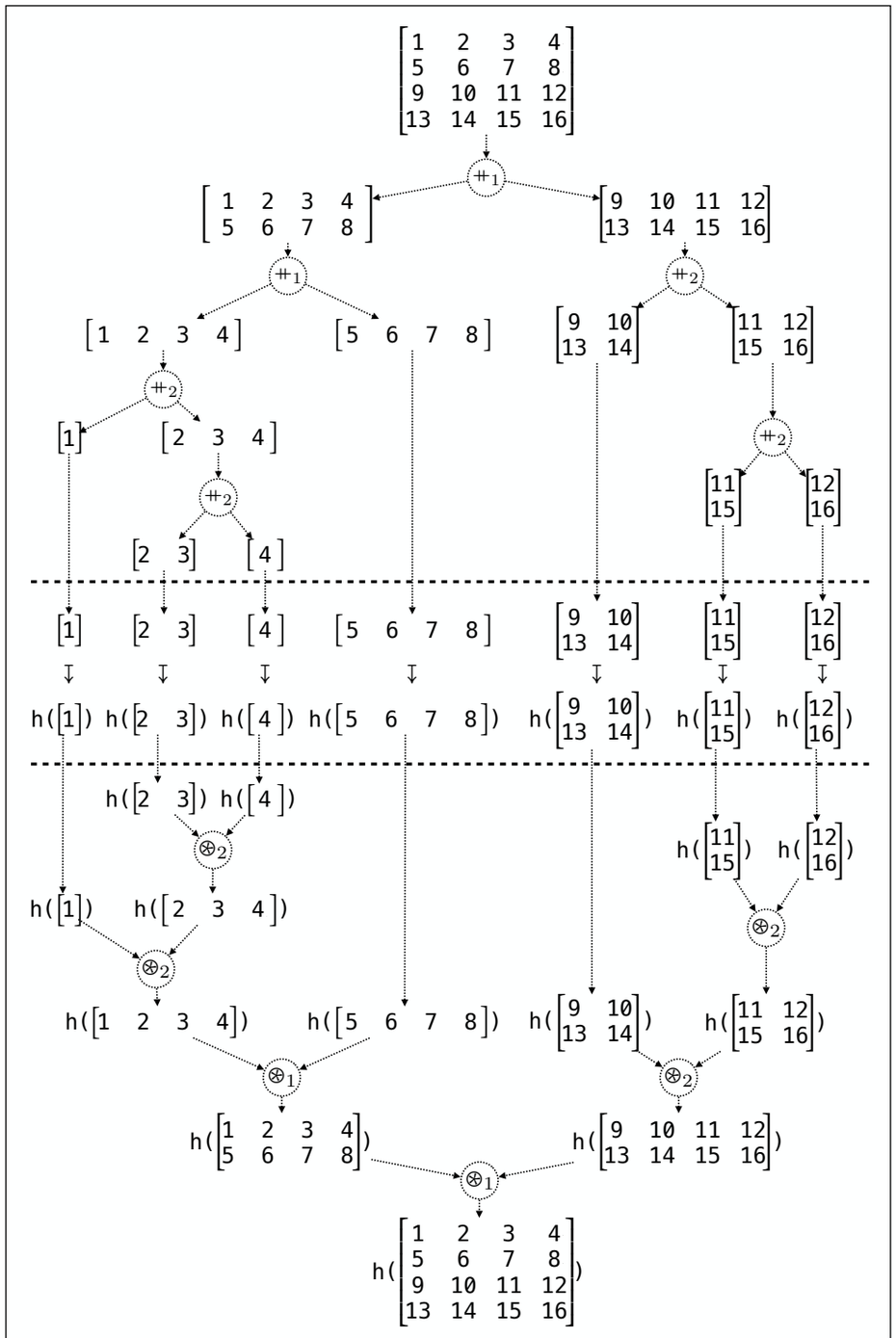


Fig. 10. MDH property recursively applied to a two-dimensional example computation

Definition 3 (Multi-Dimensional Homomorphism). Let $T^{\text{INP}}, T^{\text{OUT}} \in \texttt{TYPE}$ be two arbitrary scalar types, $D \in \mathbb{N}$ a natural number, and $\Rightarrow^{1}_{\texttt{MDA}}, \ldots, \Rightarrow^{D}_{\texttt{MDA}} : \texttt{MDA-IDX-SETs} \to \texttt{MDA-IDX-SETs}$ functions on MDA index sets. Let further $+\!\!+_d := +\!\!+^{<T^{\text{INP}} \mid D \mid d>} \in \texttt{CO}^{<id \mid T^{\text{INP}} \mid D \mid d>}$ denote concatenation (Definition 1) in dimension $d \in [1, D]_{\mathbb{N}}$ on D-dimensional MDAs that have scalar type T^{INP}.

A function

$$h^{<I_1,\ldots,I_D \in \texttt{MDA-IDX-SETs}>} : T^{\text{INP}}[\, I_1, \ldots, I_D \,] \to T^{\text{OUT}}[\, \Rightarrow^{1}_{\texttt{MDA}}(I_1), \ldots, \Rightarrow^{D}_{\texttt{MDA}}(I_D) \,]$$

is a *Multi-Dimensional Homomorphism (MDH)* that has *input scalar type* T^{INP}, *output scalar type* T^{OUT}, *dimensionality* D, and *index set functions* $\Rightarrow^{1}_{\texttt{MDA}}, \ldots, \Rightarrow^{D}_{\texttt{MDA}}$, iff for each $d \in [1, D]_{\mathbb{N}}$, there exists a combine operator $\circledast_d \in \texttt{CO}^{<\Rightarrow^{d}_{\texttt{MDA}} \mid T^{\text{OUT}} \mid D \mid d>}$ (Definition 2), such that for any concatenated input MDA $\mathfrak{a}_1 +\!\!+_d \mathfrak{a}_2$ in dimension d, the *homomorphic property* is satisfied:

$$h(\, \mathfrak{a}_1 +\!\!+_d \mathfrak{a}_2 \,) \;=\; h(\mathfrak{a}_1) \circledast_d h(\mathfrak{a}_2)$$

We denote the type of MDHs concisely as $\texttt{MDH}^{<T^{\text{INP}}, T^{\text{OUT}} \mid D \mid (\Rightarrow^{d}_{\texttt{MDA}})_{d \in [1,D]_{\mathbb{N}}}>}$.

Definition 5 (Buffer). Let $T \in \texttt{TYPE}$ be an arbitrary scalar type, $D \in \mathbb{N}_0$ a natural number[9], and $N := (N_1, \ldots, N_D) \in \mathbb{N}^D$ a sequence of natural numbers.

A *Buffer (BUF)* $\mathfrak{b}$ that has *dimensionality* D, *size* N, and *scalar type* T is a function with the following signature:

$$\mathfrak{b} : [0, N_1)_{\mathbb{N}_0} \times \ldots \times [0, N_D)_{\mathbb{N}_0} \to T \cup \{\bot\}$$

Here, $\bot$ denotes the *undefined* value. We refer to $[0, N_1)_{\mathbb{N}_0} \times \ldots \times [0, N_D)_{\mathbb{N}_0} \to T \cup \{\bot\}$ as the *type* of BUF $\mathfrak{b}$, which we also denote as $T^{N_1 \times \ldots \times N_D}$, and we refer to set $\texttt{BUF-IDX-SETs} := \{\, [0, N)_{\mathbb{N}_0} \mid N \in \mathbb{N} \,\}$ as *BUF index sets*. Analogously to Notation 1, we write $\mathfrak{b}[\, i_1, \ldots, i_D \,]$ instead of $\mathfrak{b}(i_1, \ldots, i_D)$ to avoid a too heavy usage of parentheses.

Definition 2 (Combine Operator). Let $\texttt{MDA-IDX-SETs} \dot{\times} \texttt{MDA-IDX-SETs} := \{\, (P, Q) \in \texttt{MDA-IDX-SETs} \times \texttt{MDA-IDX-SETs} \mid P \cap Q = \varnothing \,\}$ denote the set of all pairs of MDA index sets that are disjoint. Let further $\Rightarrow^{\texttt{MDA}}_{\texttt{MDA}} : \texttt{MDA-IDX-SETs} \to \texttt{MDA-IDX-SETs}$ be a function on MDA index sets, $T \in \texttt{TYPE}$ a scalar type, $D \in \mathbb{N}$ an MDA dimensionality, and $d \in [1, D]_{\mathbb{N}}$ an MDA dimension.

We refer to any binary function $\circledast$ of type (parameters in angle brackets are type parameters)

$$\circledast^{<(I_1,\ldots,I_{d-1},I_{d+1},\ldots,I_D) \in \texttt{MDA-IDX-SETs}^{D-1},\ (P,Q) \in \texttt{MDA-IDX-SETs} \dot{\times} \texttt{MDA-IDX-SETs}>} :$$

$$T[\, I_1, \ldots, \underbrace{\Rightarrow^{\texttt{MDA}}_{\texttt{MDA}}(P)}_{\uparrow d}, \ldots, I_D \,] \times T[\, I_1, \ldots, \underbrace{\Rightarrow^{\texttt{MDA}}_{\texttt{MDA}}(Q)}_{\uparrow d}, \ldots, I_D \,] \to T[\, I_1, \ldots, \underbrace{\Rightarrow^{\texttt{MDA}}_{\texttt{MDA}}(P \uplus Q)}_{\uparrow d}, \ldots, I_D \,]$$

as *combine operator* that has *index set function* $\Rightarrow^{\texttt{MDA}}_{\texttt{MDA}}$, *scalar type* T, *dimensionality* D, and *operating dimension* d. We denote combine operator's type concisely as $\texttt{CO}^{<\Rightarrow^{\texttt{MDA}}_{\texttt{MDA}} \mid T \mid D \mid d>}$.

… (>130 pages)

All concepts are fully formally defined in the MDH paper (arXiv version)

MDH: Experimental Results

MDH is experimentally evaluated in terms of ***Performance*** & ***Portability*** & ***Productivity***:

Competitors:

1. Scheduling Approach:
   - Apache TVM [1] (GPU & CPU)
2. Polyhedral Compilers:
   - PPCG [2] (GPU)
   - Pluto [3] (CPU)
3. Functional Approach:
   - Lift [4] (GPU & CPU)
4. Domain-Specific Libraries:
   - NVIDIA cuBLAS & cuDNN (GPU)
   - Intel oneMKL & oneDNN (CPU)

Case Studies:

1. Linear Algebra Routines:
   - Matrix Multiplication (`MatMul`)
   - Matrix-Vector Multiplication (`MatVec`)
2. Stencil Computations:
   - Jacobi Computation (`Jacobi1D`)
   - Gaussian Convolution (`Conv2D`)
3. Quantum Chemistry:
   - Coupled Cluster (`CCSD(T)`)
4. Data Mining:
   - Probabilistic Record Linkage (PRL)
5. Deep Learning:
   - Multi-Channel Convolution (`MCC`)
   - Capsule-Style Convolution (`MCC_Capsule`)

[1] Chen et al., “TVM: An Automated End-to-End Optimizing Compiler for Deep Learning”, OSDI’18

[2] Verdoolaege et al., “Polyhedral Parallel Code Generation for CUDA”, TACO’13

[3] Bondhugula et al., “PLuTo: A Practical and Fully Automatic Polyhedral Program Optimization System”, PLDI’08

[4] Steuwer et al., “Generating Performance Portable Code using Rewrite Rules”, ICFP’15

MDH: Experimental Results

Portability **&** ***Productivity*** **evaluated in the paper**

Highlights only

Performance Evaluation: (via runtime comparison)

Deep Learning	NVIDIA Ampere GPU									
	ResNet-50				VGG-16				MobileNet	
	Training		Inference		Training		Inference		Training	Inference
	MCC	MatMul	MCC	MatMul	MCC	MatMul	MCC	MatMul	MCC	MCC
TVM+Ansor	1.00	1.26	1.05	2.22	0.93	1.42	0.88	1.14	0.94	1.00
PPCG	3456.16	8.26	–	7.89	1661.14	7.06	5.77	5.08	2254.67	7.55
PPCG+ATF	3.28	2.58	13.76	5.44	4.26	3.92	9.46	3.73	3.31	10.71
cuDNN	0.92	–	1.85	–	1.22	–	1.94	–	1.81	2.14
cuBLAS	–	1.58	–	2.67	–	0.93	–	1.04	–	–
cuBLASEx	–	1.47	–	2.56	–	0.92	–	1.02	–	–
cuBLASLt	–	1.26	–	1.22	–	0.91	–	1.01	–	–

NVIDIA

NVCC vs NVRTC

MDH speedup over

- TVM: 0.88x – 2.22x
- PPCG: 2.58x – 13.76x
- *(cuBLAS/cuDNN: 0.91x – 2.67x)*

Deep Learning	Intel Skylake CPU									
	ResNet-50				VGG-16				MobileNet	
	Training		Inference		Training		Inference		Training	Inference
	MCC	MatMul	MCC	MatMul	MCC	MatMul	MCC	MatMul	MCC	MCC
TVM+Ansor	1.53	1.05	1.14	1.20	1.97	1.14	2.38	1.27	3.01	1.40
Pluto	355.81	49.57	364.43	13.93	130.80	93.21	186.25	36.30	152.14	75.37
Pluto+ATF	13.08	19.70	170.69	6.57	3.11	6.29	53.61	8.29	3.50	25.41
oneDNN	0.39	–	5.07	–	1.22	–	9.01	–	1.05	4.20
oneMKL	–	0.44	–	1.09	–	0.88	–	0.53	–	–
oneMKL(JIT)	–	6.43	–	8.33	–	27.09	–	9.78	–	–

intel

MDH speedup over

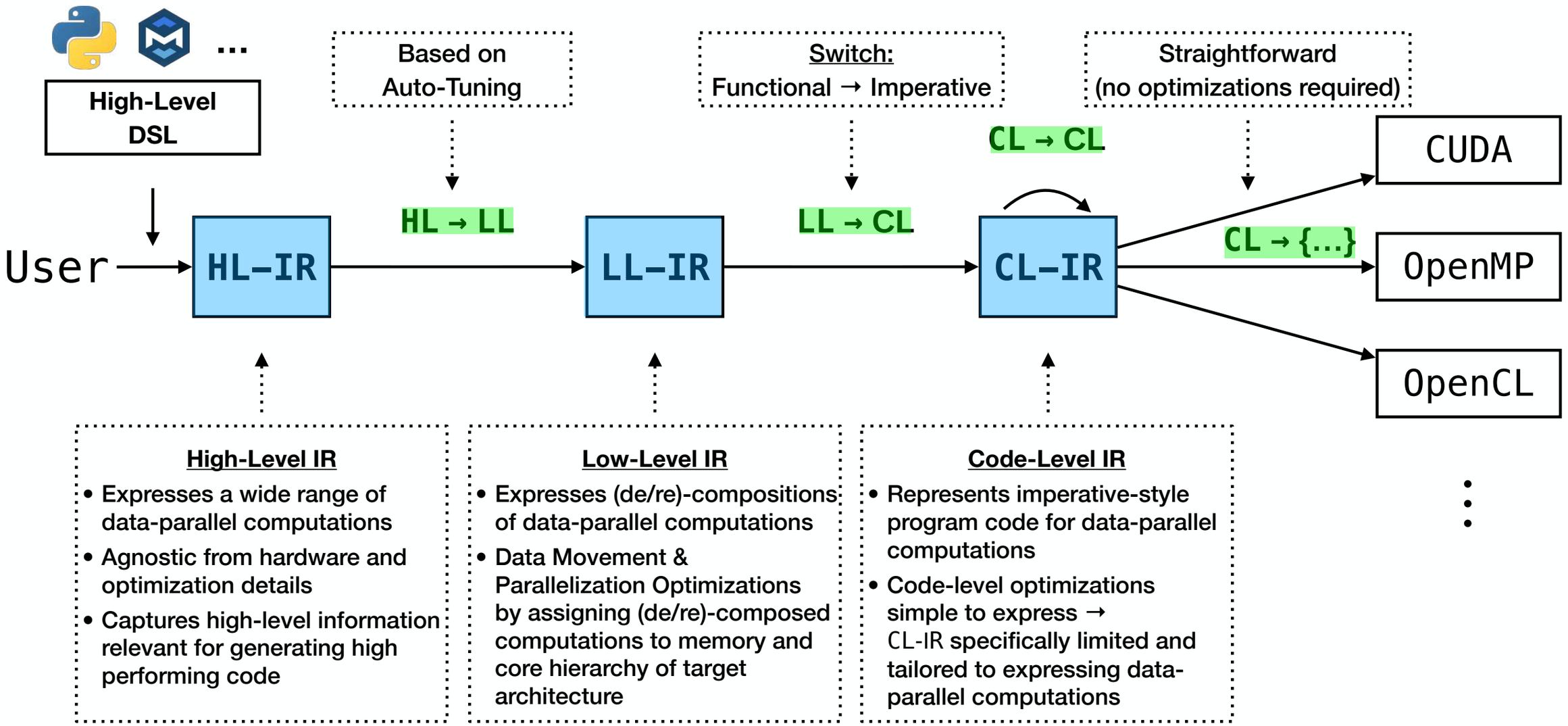
- TVM: 1.05 – 3.01x
- Pluto: 6.29x – 364.43x
- *(oneMKL/oneDNN: 0.39x – 9.01x)*

Case Study "Deep Learning" for which most competitors are highly optimized (most challenging for us!)

Significantly higher speedups for other case studies,
e.g., >170x over TVM on GPU already for straightforward dot products

MDH: Code Generation

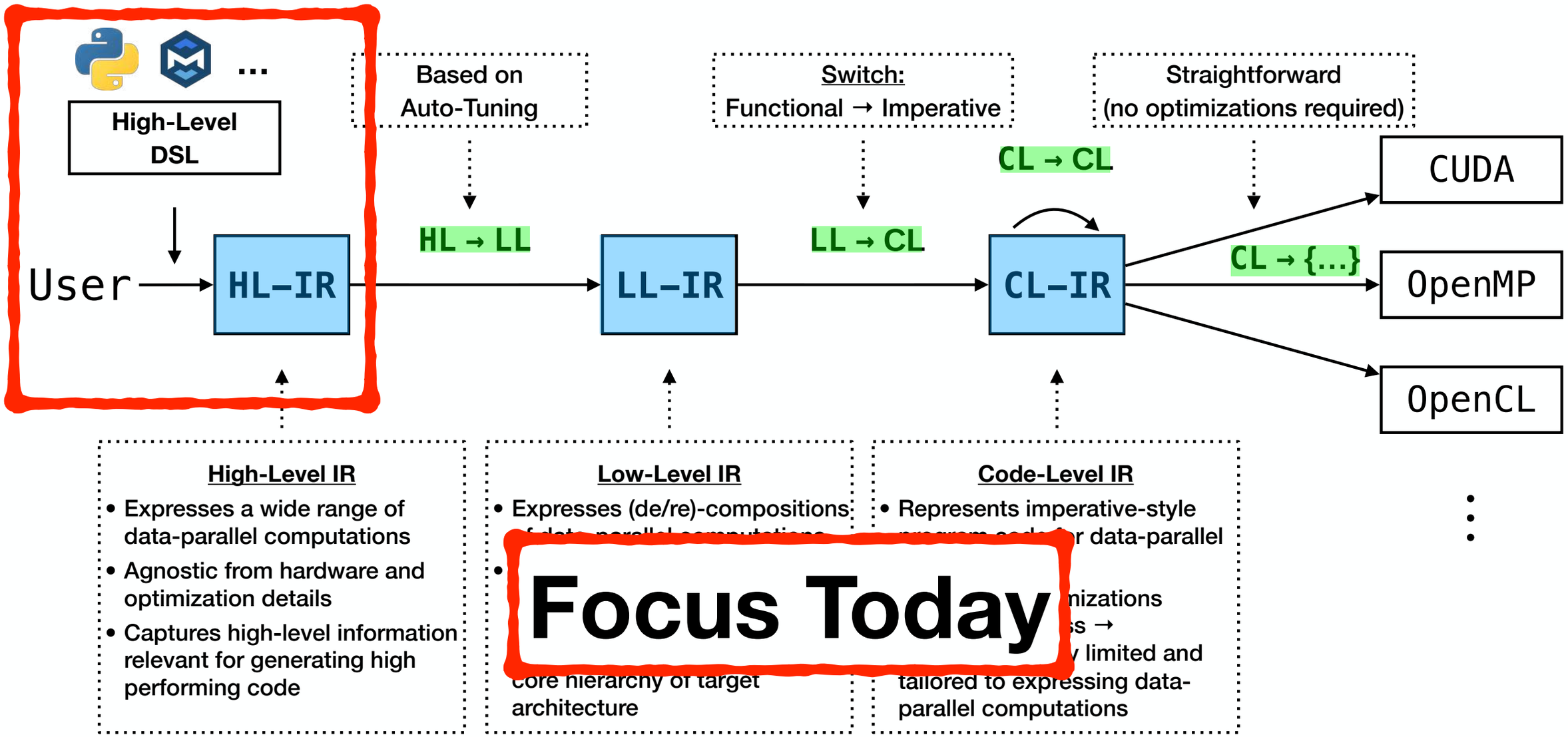
WIP



We rely on a *structured* code generation process, based on IRs and IR-Transformations

MDH: Code Generation

WIP



We rely on a *structured* code generation process, based on IRs and IR-Transformations

MDH: High-Level Representation

Quick Dive

We provide a **Python interface** for MDH's high-level program representation:

MDH

```
MatVec<T∈TYPE|I,K∈ℕ> := out_view<T>( w:(i,k)↦(i) ) ∘
                         md_hom<I,K>( *, (#,+) ) ∘
                          inp_view<T,T>( M:(i,k)↦(i,k) , v:(i,k)↦(k) )
```

in MDH Formalism

MatVec

```python
def matvec( T: BasicType, I: int, K: int ):

    @mdh()
    def mdh_matvec():
        return (
              out_view[T]( w = [lambda i,k: (i)] ),
                md_hom[I,K]( mul, ( cc, pw(scalar_plus) ) ),
                  inp_view[T,T]( M = [lambda i,k: (i,k)] ,
                                 v = [lambda i,k: (k)  ] )
         )
```

The MDH-Python-Interface is designed to be very close to MDH's formal representation

MDH: High-Level Representation

Quick Dive

We provide a **Python interface** for MDH's high-level program representation:

MDH

```
MatVec<T∈TYPE|I,K∈ℕ> := out_view<T>( w:(i,k)↦(i) ) ∘
                         md_hom<I,K>( *, (#,+) ) ∘
                          inp_view<T,T>( M:(i,k)↦(i,k) , v:(i,k)↦(k) )
```

in MDH Formalism

MatVec

```python
def matvec( T: BasicType, I: int, K: int ):

    @mdh()
    def mdh_matvec():
        @scalar_function()
        def mul( out: Scalar[T], inp: Scalar[T,T] ):
            out['w'] = inp['M'] * inp['v']

    return (
        out_view[T]( w = [lambda i,k: (i)] ),
          md_hom[I,K]( mul, ( cc, pw(scalar_plus) ) ),
            inp_view[T,T]( M = [lambda i,k: (i,k)] ,
                           v = [lambda i,k: (k)  ] )
     )
```

We allow custom scalar functions

MDH: High-Level Representation

Quick Dive

We provide a **Python interface** for MDH's high-level program representation:

MDH

```
MatVec<T∈TYPE|I,K∈ℕ> := out_view<T>( w:(i,k)↦(i) ) ∘
                         md_hom<I,K>( *, (#,+) ) ∘
                          inp_view<T,T>( M:(i,k)↦(i,k) , v:(i,k)↦(k) )
```

in MDH Formalism

MatVec

```python
def matvec( T: BasicType, I: int, K: int ):

    @mdh()
    def mdh_matvec():

        @pw_custom_func()
        def scalar_plus( res: Scalar[T],
                         lhs: Scalar[T], rhs: Scalar[T] ):
            res['w'] = lhs['w'] + rhs['w']

        return (
            out_view[T]( w = [lambda i,k: (i)] ),
              md_hom[I,K]( mul, ( cc, pw(scalar_plus) ) ),
                inp_view[T,T]( M = [lambda i,k: (i,k)] ,
```

We allow custom point-wise operators

MDH: High-Level Representation

Quick Dive

We provide a **Python interface** for MDH's high-level program representation:

```
MatVec^<T∈TYPE|I,K∈ℕ>  :=  out_view<T>( w:(i,k)↦(i) ) ∘                    MDH
                            md_hom<I,K>( *, (+,+) ) ∘
                                inp_view<T,T>( M:(i,k)↦(i,k) , v:(i,k)↦(k) )
```

in MDH Formalism

MatVec

```python
def matvec( T: BasicType, I: int, K: int ):

    @mdh()
    def mdh_matvec():

        def cc(T: BasicType, D: int, d: int):
            @combine_operator(
                index_set_function = lambda I: I,
                scalar_type = T,
                dimensionality = D,
                operating_dimension = d
            )
            def cc__T_D_d( I, P,Q ):
                def cc__T_D_d__I_PQ( res, lhs,rhs ):
                    for i[1, ..., d - 1] in I[1, ..., d - 1]:
                        for i[d + 1, ..., D] in I[d + 1, ..., D]:
                            for i[d] in P:
                                res[i[1, ..., d, ..., D]] = lhs[i[1, ..., d, ..., D]]
                            for i[d] in Q:
                                res[i[1, ..., d, ..., D]] = rhs[i[1, ..., d, ..., D]]
                return cc__T_D_d__I_PQ
            return cc__T_D_d

        return (
            out_view[T]( w = [lambda i,k: (i)] ),
              md_hom[I,K]( mul, ( cc, pw(scalar_plus) ) ),
                inp_view[T,T]( M = [lambda i,k: (i,k)] ,
                               v = [lambda i,k: (k)  ] )
        )
```

We allow custom combine operators

MDH: High-Level Representation

Quick Dive

We provide a **Python interface** for MDH's high-level program representation:

```
MatVec<T∈TYPE|I,K∈ℕ> := out_view<T>( w:(i,k)↦(i) ) ∘                    MDH
                         md_hom<I,K>( *, (+,+) ) ∘
                          inp_view<T,T>( M:(i,k)↦(i,k) , v:(i,k)↦(k) )
```

in MDH Formalism

MatVec

```python
def matvec(T: BasicType, I: int, K: int):

    @mdh( out( w = Buffer[T]                    ) ,
          inp( M = Buffer[T], v = Buffer[T] ) ,
          combine_ops( cc, pw(scalar-plus) )  )
    def mdh_matvec( w, M,v ):
        for i in range(I):
            for k in range(K):
                w[i] = M[i, k] * v[k]
```

MDH also takes as input
straightforward sequential Python code
(instead of DSL programs)

MDH: High-Level Representation

Quick Dive

We offer an **MLIR interface** for MDH's high-level program representation:

```
out_view<f32>( w:(i,k)↦(i) )
   md_hom<128,64>( *, (+,+) )
      inp_view<f32,f32>( M:(i,k)↦(i,k) , v:(i,k)↦(k) )
```

MDH

in MDH Formalism

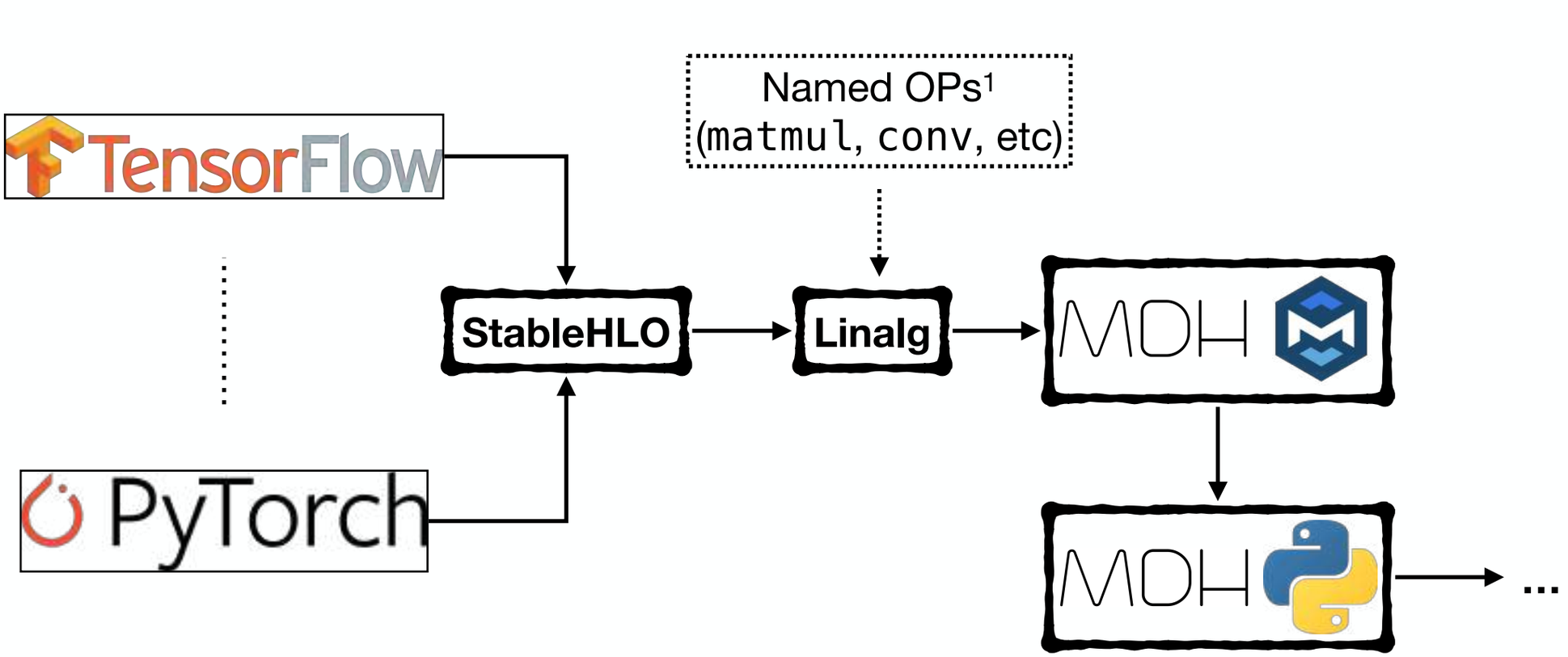
MatVec

```
func.func @main()
{
  %M = memref.alloc() : memref<128x64xf32>
  %v = memref.alloc() : memref<64xf32>

  %w = mdh.compute "mdh_matvec"
  {
    inp_view =
    [
      [ affine_map<( i,k ) -> ( i,k )> ],
      [ affine_map<( i,k ) -> ( k )  > ]
    ],

    md_hom =
    {
      scalar_func = @mul,
      combine_ops = [ "cc", ["pw",@add] ]
    },

    out_view =
    [
      [ affine_map<( i,k ) -> ( i )> ]
    ]
  }
  {
    inp_types = [ f32, f32 ],
    mda_size  = [ 128,64 ],
    out_types = [ f32 ]
  }( %M,%v ):( memref<128x64xf32>,memref<64xf32> )
               -> memref<128xf32>

  return
}
```

MLIR interface for MDH

MDH: High-Level Representation

WIP



Implemented by
Jens & Lars Hunloh

Our MLIR interface allows
easy integration of MDH into DL Frameworks

[1]We consider the MDH abstraction level to be between *Linalg's Named OPs* and *Linalg generic*.

MDH: Comaprison to MLIR Linalg

Quick Reminder: *MLIR Linalg Dialect*

Linalg

```
#map1 = affine_map<(d0, d1) -> (d0, d1)>
#map2 = affine_map<(d0, d1) -> (d1)    >
#map3 = affine_map<(d0, d1) -> (d0)    >
module {
  func.func @main() {
    %M = memref.alloc() : memref<128x64xf32>
    %v = memref.alloc() : memref<64xf32>
    %w = memref.alloc() : memref<128xf32>
    linalg.generic
    {
      indexing_maps = [#map1, #map2, #map3],
      iterator_types = ["parallel", "reduction"]
    } ins(%M,%v:memref<128x64xf32>,memref<64xf32>)
      outs(%w:memref<128xf32>) {
    ^bb0(%in_1: f32, %in_2: f32, %out: f32):
      %0 = arith.mulf %in_1, %in_2 : f32
      %1 = arith.addf %out, %0 : f32
      linalg.yield %1 : f32
    }
    return
  }
}
```

Linalg expressing MatVec

Quick reminder “`Linalg`”, before comparison “`Linalg` vs. `MDH`"

Quick Reminder: Linalg

Quick Reminder: *MLIR Linalg Dialect*

Linalg

```
#map1 = affine_map<(d0, d1) -> (d0, d1)>
#map2 = affine_map<(d0, d1) -> (d1)    >
#map3 = affine_map<(d0, d1) -> (d0)    >
module {
  func.func @main() {
    %M = memref.alloc() : memref<128x64xf32>
    %v = memref.alloc() : memref<64xf32>
    %w = memref.alloc() : memref<128xf32>
    linalg.generic
    {
      indexing_maps = [#map1, #map2, #map3],
      iterator_types = ["parallel", "reduction"]
    } ins(%M,%v:memref<128x64xf32>,memref<64xf32>)
      outs(%w:memref<128xf32>) {
    ^bb0(%in_1: f32, %in_2: f32, %out: f32):
      %0 = arith.mulf %in_1, %in_2 : f32
      %1 = arith.addf %out, %0 : f32
      linalg.yield %1 : f32
    }
    return
  }
}
```

Linalg expressing MatVec

Accesses to Input/Output Data

Quick Reminder: Linalg

Quick Reminder: *MLIR Linalg Dialect*

Linalg

```
#map1 = affine_map<(d0, d1) -> (d0, d1)>
#map2 = affine_map<(d0, d1) -> (d1)    >
#map3 = affine_map<(d0, d1) -> (d0)    >
module {
  func.func @main() {
    %M = memref.alloc() : memref<128x64xf32>
    %v = memref.alloc() : memref<64xf32>
    %w = memref.alloc() : memref<128xf32>
    linalg.generic
    {
      indexing_maps = [#map1, #map2, #map3],
      iterator_types = ["parallel", "reduction"]
    } ins(%M,%v:memref<128x64xf32>,memref<64xf32>)
      outs(%w:memref<128xf32>) {
    ^bb0(%in_1: f32, %in_2: f32, %out: f32):
      %0 = arith.mulf %in_1, %in_2 : f32
      %1 = arith.addf %out, %0 : f32
      linalg.yield %1 : f32
    }
    return
  }
}
```

Linalg expressing MatVec

Iteration Space Specification

Quick Reminder: Linalg

Quick Reminder: *MLIR Linalg Dialect*

Linalg

```
#map1 = affine_map<(d0, d1) -> (d0, d1)>
#map2 = affine_map<(d0, d1) -> (d1)    >
#map3 = affine_map<(d0, d1) -> (d0)    >
module {
  func.func @main() {
    %M = memref.alloc() : memref<128x64xf32>
    %v = memref.alloc() : memref<64xf32>
    %w = memref.alloc() : memref<128xf32>
    linalg.generic
    {
      indexing_maps = [#map1, #map2, #map3],
      iterator_types = ["parallel", "reduction"]
    } ins(%M,%v:memref<128x64xf32>,memref<64xf32>)
      outs(%w:memref<128xf32>) {
    ^bb0(%in_1: f32, %in_2: f32, %out: f32):
      %0 = arith.mulf %in_1, %in_2 : f32
      %1 = arith.addf %out, %0 : f32
      linalg.yield %1 : f32
    }
    return
  }
}
```

Linalg expressing MatVec

MatVec Computation (`mul` and `add`)

Comparison: Linalg vs MDH

Linalg

```
#map1 = affine_map<(d0, d1) -> (d0, d1)>
#map2 = affine_map<(d0, d1) -> (d1)    >
#map3 = affine_map<(d0, d1) -> (d0)    >
module {
  func.func @main() {
    %M = memref.alloc() : memref<128x64xf32>
    %v = memref.alloc() : memref<64xf32>
    %w = memref.alloc() : memref<128xf32>
    linalg.generic
    {
      indexing_maps = [#map1, #map2, #map3],
      iterator_types = ["parallel", "reduction"]
    } ins(%M,%v:memref<128x64xf32>,memref<64xf32>)
      outs(%w:memref<128xf32>) {
    ^bb0(%in_1: f32, %in_2: f32, %out: f32):
      %0 = arith.mulf %in_1, %in_2 : f32
      %1 = arith.addf %out, %0 : f32
      linalg.yield %1 : f32
    }
    return
  }
}
```

MDH

```
func.func @main()
{
  %M = memref.alloc() : memref<128x64xf32>
  %v = memref.alloc() : memref<64xf32>

  %w = mdh.compute "mdh_matvec"
  {
    inp_view =
    [
      [ affine_map<( i,k ) -> ( i,k )> ],
      [ affine_map<( i,k ) -> ( k )  > ]
    ],

    md_hom =
    {
      scalar_func = @mul,
      combine_ops = [ "cc", ["pw",@add] ]
    },

    out_view =
    [
      [ affine_map<( i,k ) -> ( i )> ]
    ]
  }
  {
    inp_types = [ f32, f32 ],
    mda_size  = [ 128,64 ],
    out_types = [ f32 ]
  }( %M,%v ):( memref<128x64xf32>,memref<64xf32> )
               -> memref<128xf32>

  return
}
```

Comparison: Linalg vs MDH

Linalg

```
#map1 = affine_map<(d0, d1) -> (d0, d1)>
#map2 = affine_map<(d0, d1) -> (d1)    >
#map3 = affine_map<(d0, d1) -> (d0)    >
module {
  func.func @main() {
    %M = memref.alloc() : memref<128x64xf32>
    %v = memref.alloc() : memref<64xf32>
    %w = memref.alloc() : memref<128xf32>
    linalg.generic
    {
      indexing_maps = [#map1, #map2, #map3],
      iterator_types = ["parallel", "reduction"]
    } ins(%M,%v:memref<128x64xf32>,memref<64xf32>)
      outs(%w:memref<128xf32>) {
    ^bb0(%in_1: f32, %in_2: f32, %out: f32):
      %0 = arith.mulf %in_1, %in_2 : f32
      %1 = arith.addf %out, %0 : f32
      linalg.yield %1 : f32
    }
    return
  }
}
```

MDH

```
func.func @main()
{
  %M = memref.alloc() : memref<128x64xf32>
  %v = memref.alloc() : memref<64xf32>

  %w = mdh.compute "mdh_matvec"
  {
    inp_view =
    [
      [ affine_map<( i,k ) -> ( i,k )> ],
      [ affine_map<( i,k ) -> ( k )  > ]
    ],

    md_hom =
    {
      scalar_func = @mul,
      combine_ops = [ "cc", ["pw",@add] ]
    },

    out_view =
    [
      [ affine_map<( i,k ) -> ( i )> ]
    ]
  }
  {
    inp_types = [ f32, f32 ],
    mda_size  = [ 128,64 ],
    out_types = [ f32 ]
  }( %M,%v ):( memref<128x64xf32>,memref<64xf32> )
              -> memref<128xf32>

  return
}
```

Significant design difference:

MDH **separates** & **explicitly captures** the **scalar operation** (e.g., `mul`) and **operations for combining intermediate results** (e.g., add)

Note: MatVec is a simple example!

Comparison: Linalg vs MDH

Advantages we see in MDH Design:

1. **Performance:** parallelizing & optimizing also reduction-based (sub-)computations within the target computation

MDH can parallelize & optimize also 2nd dimension ($\circledast_2$)

$$\begin{pmatrix} M_{1,1} & \dots & M_{1,K} \\ \vdots & \ddots & \vdots \\ M_{I,1} & \dots & M_{I,K} \end{pmatrix}, \begin{pmatrix} v_1 \\ \vdots \\ v_K \end{pmatrix} \overset{\texttt{MatVec}}{\mapsto} \begin{pmatrix} f(M_{1,1}, v_1) & \dots & f(M_{1,K}, v_K) \\ \vdots & \ddots & \vdots \\ f(M_{I,1}, v_1) & \dots & f(M_{I,K}, v_K) \end{pmatrix} \begin{matrix} \xrightarrow{\circledast_2} \\ \downarrow \circledast_1 \end{matrix} = \begin{pmatrix} M_{1,1} * v_1 + \ldots + M_{1,K} * v_K \\ \vdots \\ M_{I,1} * v_1 + \ldots + M_{I,K} * v_K \end{pmatrix} = \begin{pmatrix} w_1 \\ \vdots \\ w_I \end{pmatrix}$$

`Matrix-Vector Multiplication`

Linalg struggles with parallelizing and optimizing reduction-heavy computations

Comparison: Linalg vs MDH

Advantages we see in MDH Design:

2. **Naturality:** i) avoiding *unnecessary memory accesses* (e.g., Linalg requires 0-initialized output vector for MatVec) and ii) *not requiring existence of neutral elements* for combine ops

Linalg

```
// ...
module {
  func.func @main() {
    %M = memref.alloc() : memref<128x64xf32>
    %v = memref.alloc() : memref<64xf32>
    %w = memref.alloc() : memref<128xf32>
    linalg.generic
    { /* ... */ }
      ins(%M,%v:memref<128x64xf32>,memref<64xf32>)
      outs(%w:memref<128xf32>)
    { /* ... */ }
    return
  }
}
```

Linalg cannot express w=M∗v, instead of w+=M∗v?

Needs existence and initialization with neutral element of combine ops

Comparison: Linalg vs MDH

Advantages we see in MDH Design:

3. **Expressivity:** expressing also more advanced computations (whose reduction dimensions rely on different kinds of operators)

```
#parallel for reduce ⊕1
for( … ) {
  #parallel for reduce ⊕2
  for( … ) {
    // …
    out_2 ⊕2= foo(…)
  }
  out_1 ⊕1= out_2;
}
```

Intermediate results of loops are combined using different combine operators (e.g., MBBS example [3])

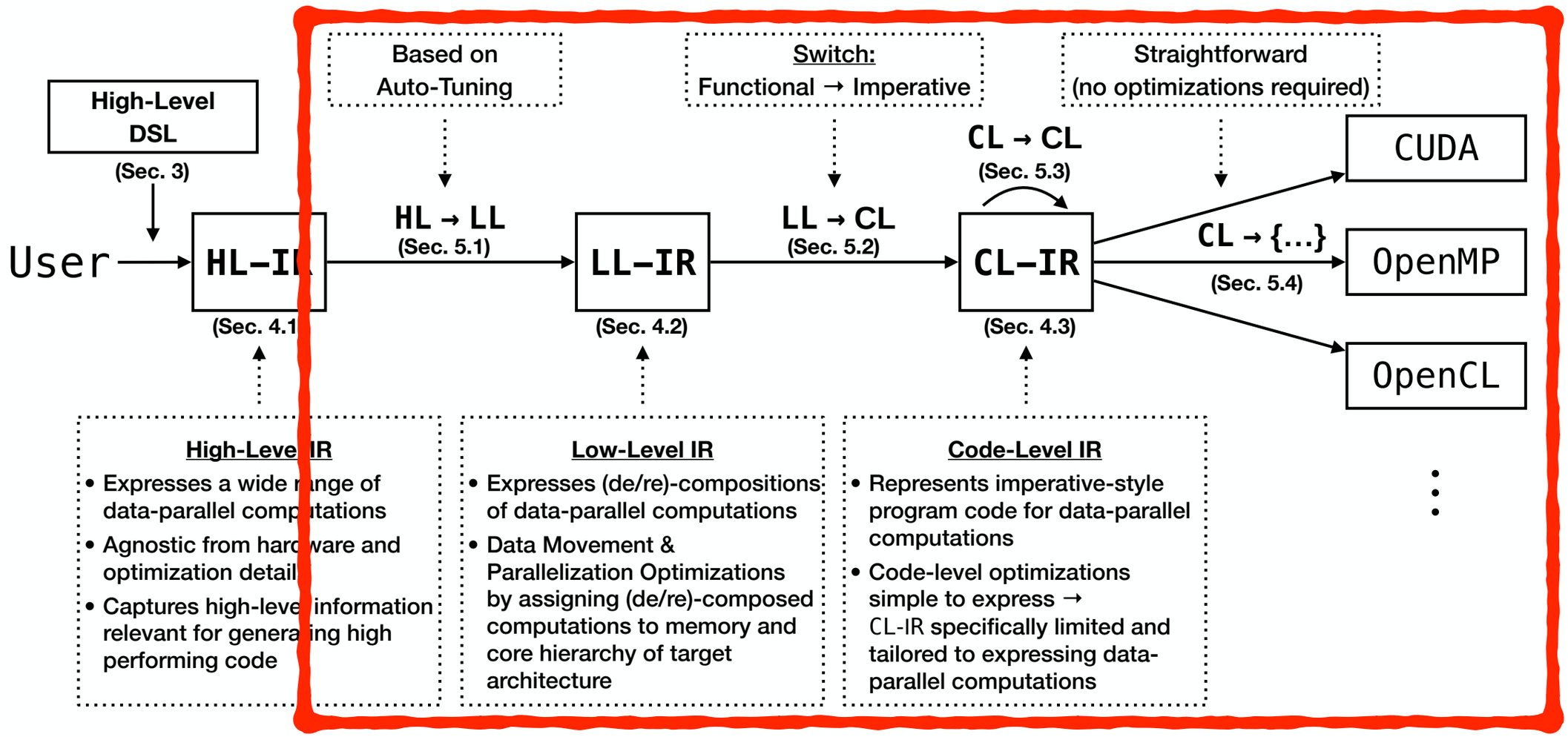
Farzan, Nicolet,“Modular Divide-and-Conquer Parallelization of Nested Loops”, **PLDI19**

Linalg cannot express computations relying on different kinds of combine operators

Comparison: Linalg vs MDH

Advantages we see in MDH Design:

4. **Code Generation:** MDH is more than a representation → the *major part* of MDH is about describing how *optimized* code is generated from its HL representation



Linalg leaves open the (non-trivial) challenge of code generation (for GPU, CPU, …)

Comparison: Linalg vs MDH

Advantages we see in MDH Design:

5. **Formalized:** MDH is fully formalized (including its optimization space and lowering process)

Advantages of *Formalism*:

Formally based *error checking* and precise *error messages*

***ML-Based* search space exploration, based on formalism**

Guaranteeing *structured & correct* code generation

...

Linalg is not formalized

Comparison: Linalg vs MDH

Further Observation: *iteration space sizes* not requested in Linalg → *convenient*, but *narrowed*

```
…
module {
  func.func @main() {
    …
    linalg.generic
    {…}
      ins(%M,%v:memref<128x64xf32>,memref<64xf32>)
      outs(%w:memref<128xf32>) {
    ^bb0(…):
      …
    }
    return
  }
}
```

vs.

```
func.func @main()
{
  …
  %w = mdh.compute "mdh_matvec"
  {
    inp_view = …
    md_hom   = …
    out_view = …
 }
  {
    inp_types = [ f32, f32 ],
    mda_size  = [ 128,64 ],
    out_types = [ f32 ]
  }( %M,%v ):( memref<128x64xf32>,memref<64xf32> )
                -> memref<128xf32>
  …
}
```

Final Remark about "Linalg vs. MDH":

Exploiting MDH design for code generation is complex, but elaborated and (formally) explained in [1]

[1] "(De/Re)-Composition of Data-Parallel Computations via Multi Dimensional Homomorphisms", **[TOPLAS 2024]**

tqchen — Feb '20

(also observed in TVM community)

This is a restriction in the current tensor expression language, because reduction is quite complicated to be processed in nested form.

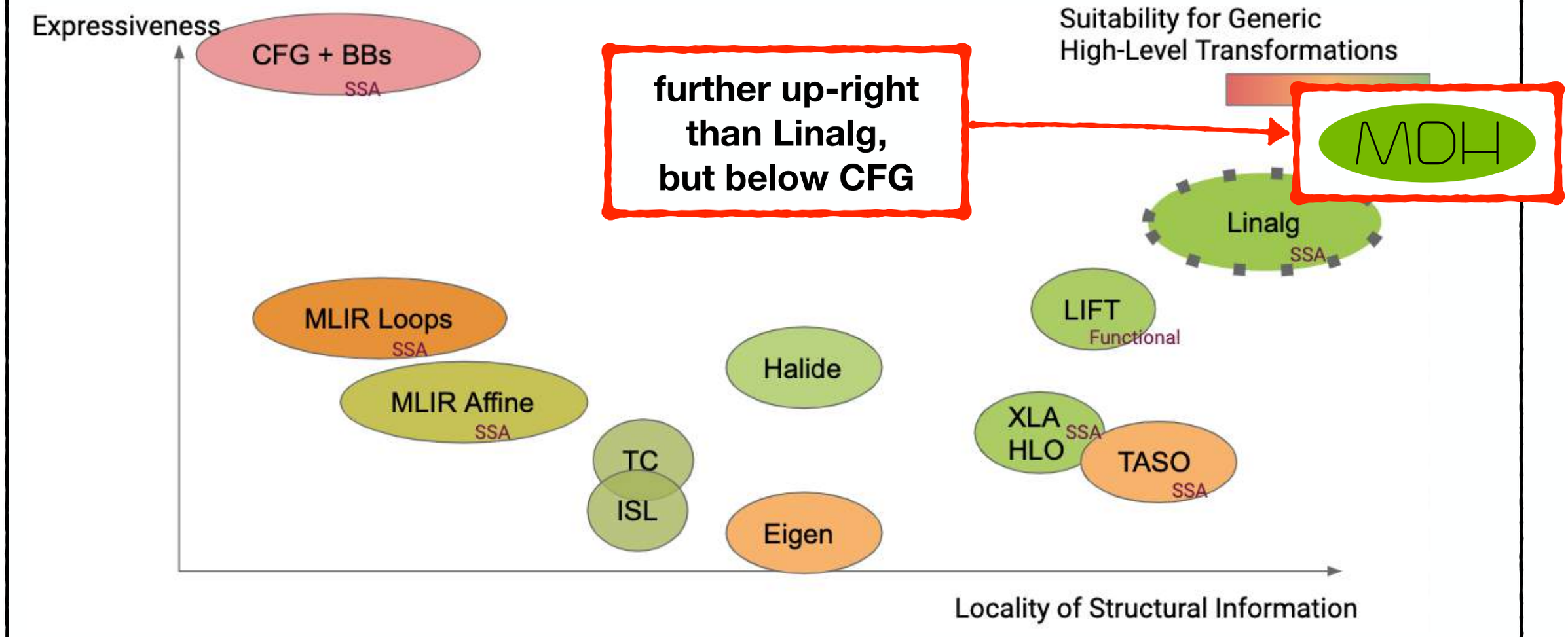
There are ongoing effort to enhance low-level IR passes to enable more powerful tensorization, which hopefully will resolve the issue you raised.

Comparison: Linalg vs MDH

Linalg classification figure (from *here*):

Summary of Existing Alternatives a Picture

Lastly, we summarize our observations of lessons from Prior Art—when viewed under the lense of our Core Guiding Principles — with the following picture.



This figure is not meant to be perfectly accurate but a rough map of how we view the distribution of structural information in existing systems, from a codegen-friendly angle. Unsurprisingly, the Linalg Dialect and its future evolutions aspire to a position in the top-right of this map.

"Linalg Dialect Rationale: The Case For Compiler-Friendly Custom Operations"

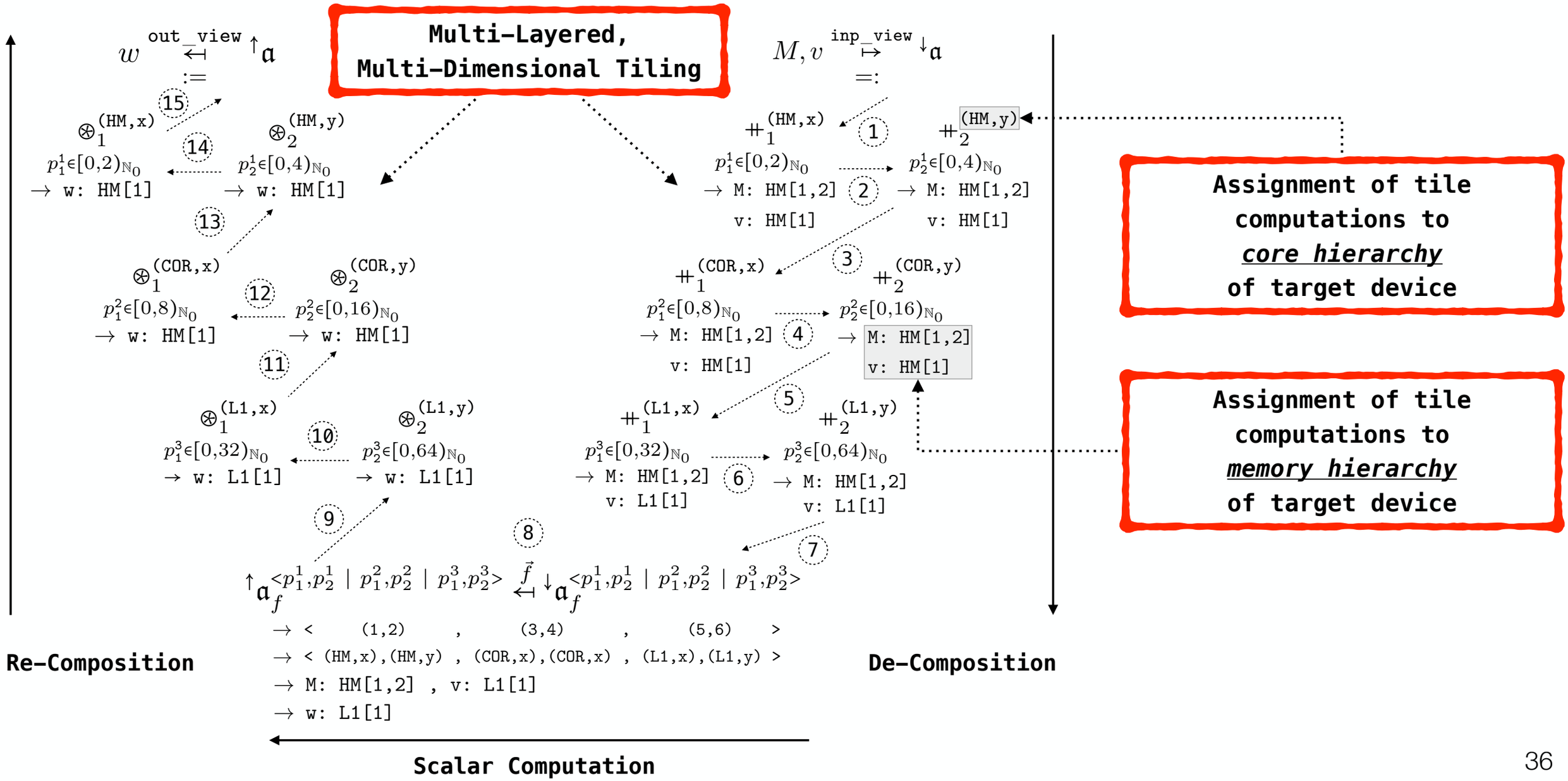
MDH seems more structured & expressive than Linalg

MDH: Low-Level Representation

Basic Idea

Goals:

1. Expressing a hardware- & data-optimized *de-composition* and *re-composition* of data-parallel computations, based on an *Abstract System Model (ASM)*
2. Being straightforwardly transformable to executable program code (e.g., in OpenMP, CUDA, and OpenCL) — major optimization decisions explicitly expressed in low-level representation



MDH: Lowering: *High Level → Low-Level*

Based on (formally defined) performance-critical parameters, for a *structured* optimization process:

No.	Name	Range	Description
0	#PRT	MDH-LVL $\to \mathbb{N}$	number of parts
D1	$\sigma_{\downarrow\text{-ord}}$	MDH-LVL $\twoheadrightarrow$ MDH-LVL	de-composition order
D2	$\leftrightarrow_{\downarrow\text{-ass}}$	MDH-LVL $\twoheadrightarrow$ ASM-LVL	ASM assignment (de-composition)
D3	$\downarrow\text{-mem}^{\langle ib \rangle}$	MDH-LVL $\to$ MR	memory regions of input BUFs (ib)
D4	$\sigma^{\langle ib \rangle}_{\downarrow\text{-mem}}$	MDH-LVL $\to [1,\ldots,D^{\text{IB}}_{\text{ib}}]_{\mathcal{S}}$	memory layouts of input BUFs (ib)
S1	$\sigma_{f\text{-ord}}$	MDH-LVL $\twoheadrightarrow$ MDH-LVL	scalar function order
S2	$\leftrightarrow_{f\text{-ass}}$	MDH-LVL $\twoheadrightarrow$ ASM-LVL	ASM assignment (scalar function)
S3	$f^{\downarrow}\text{-mem}^{\langle ib \rangle}$	MR	memory region of input BUF (ib)
S4	$\sigma^{\langle ib \rangle}_{f^{\downarrow}\text{-mem}}$	$[1,\ldots,D^{\text{IB}}_{\text{ib}}]_{\mathcal{S}}$	memory layout of input BUF (ib)
S5	$f^{\uparrow}\text{-mem}^{\langle ob \rangle}$	MR	memory region of output BUF (ob)
S6	$\sigma^{\langle ob \rangle}_{f^{\uparrow}\text{-mem}}$	$[1,\ldots,D^{\text{OB}}_{\text{ob}}]_{\mathcal{S}}$	memory layout of output BUF (ob)
R1	$\sigma_{\uparrow\text{-ord}}$	MDH-LVL $\twoheadrightarrow$ MDH-LVL	re-composition order
R2	$\leftrightarrow_{\uparrow\text{-ass}}$	MDH-LVL $\twoheadrightarrow$ ASM-LVL	ASM assignment (re-composition)
R3	$\uparrow\text{-mem}^{\langle ob \rangle}$	MDH-LVL $\to$ MR	memory regions of output BUFs (ob)
R4	$\sigma^{\langle ob \rangle}_{\uparrow\text{-mem}}$	MDH-LVL $\to [1,\ldots,D^{\text{OB}}_{\text{ob}}]_{\mathcal{S}}$	memory layouts of output BUFs (ob)

Table 1. Tuning parameters of our low-level expressions

exploiting core hierarchy (parallelization)

exploiting memory hierarchy (data movements)

...

...

...

...

Our parameters *unify* & *generalize* & *combine* (and also formalize) well-proven optimizations (e.g., tiling, data movements, and parallelization)

We use our ***Auto-Tuning Framework (ATF)*** to *automatically* determine optimized values of parameters[1]

[1] We optionally allow (expert) users to incorporate their knowledge into the optimization process via *MDH-Based Schedules* [CC'23]

MDH: WIP & Future Directions

Many promising future directions (detailed discussion available *here*):

High-Level Program Transformations (a.k.a. Fusion)

$$\texttt{md\_hom}(g,(++,\ldots,++)) \circ \texttt{md\_hom}(f,(++,\ldots,++)) \rightarrow \texttt{md\_hom}(g \circ f,(++,\ldots,++))$$

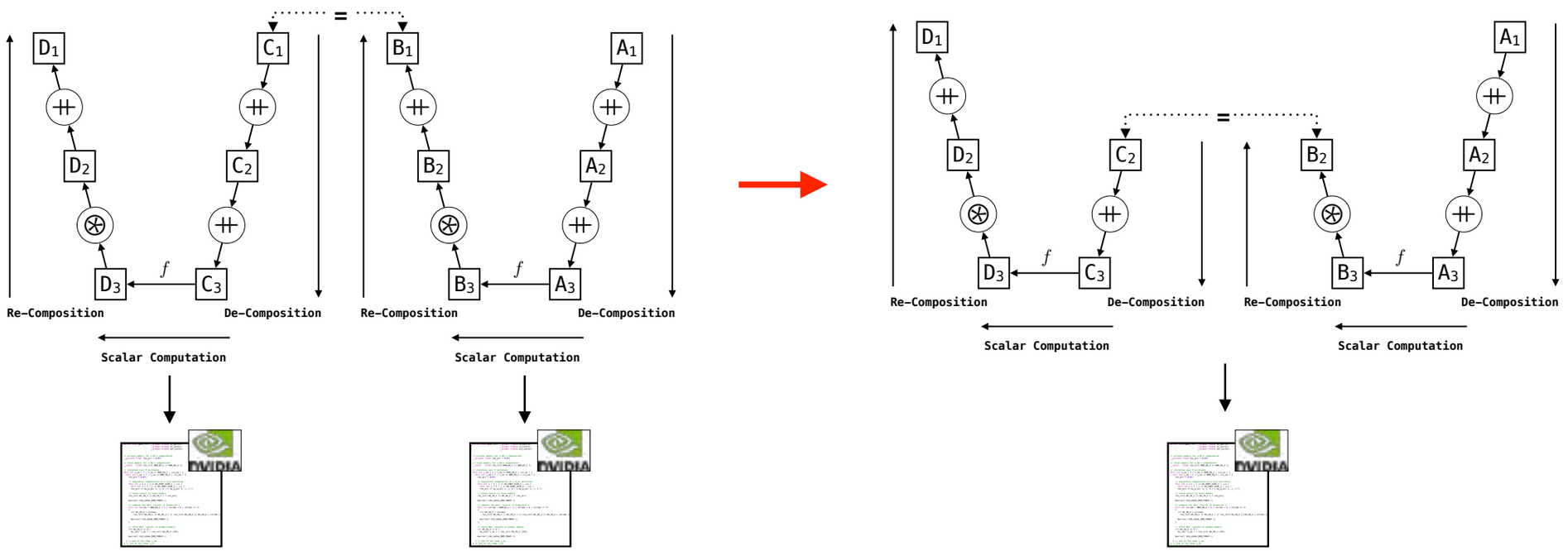
Sparse Computations

...

Domain-Specific HW

...

Low-Level Program Transformations (a.k.a. Fusion)



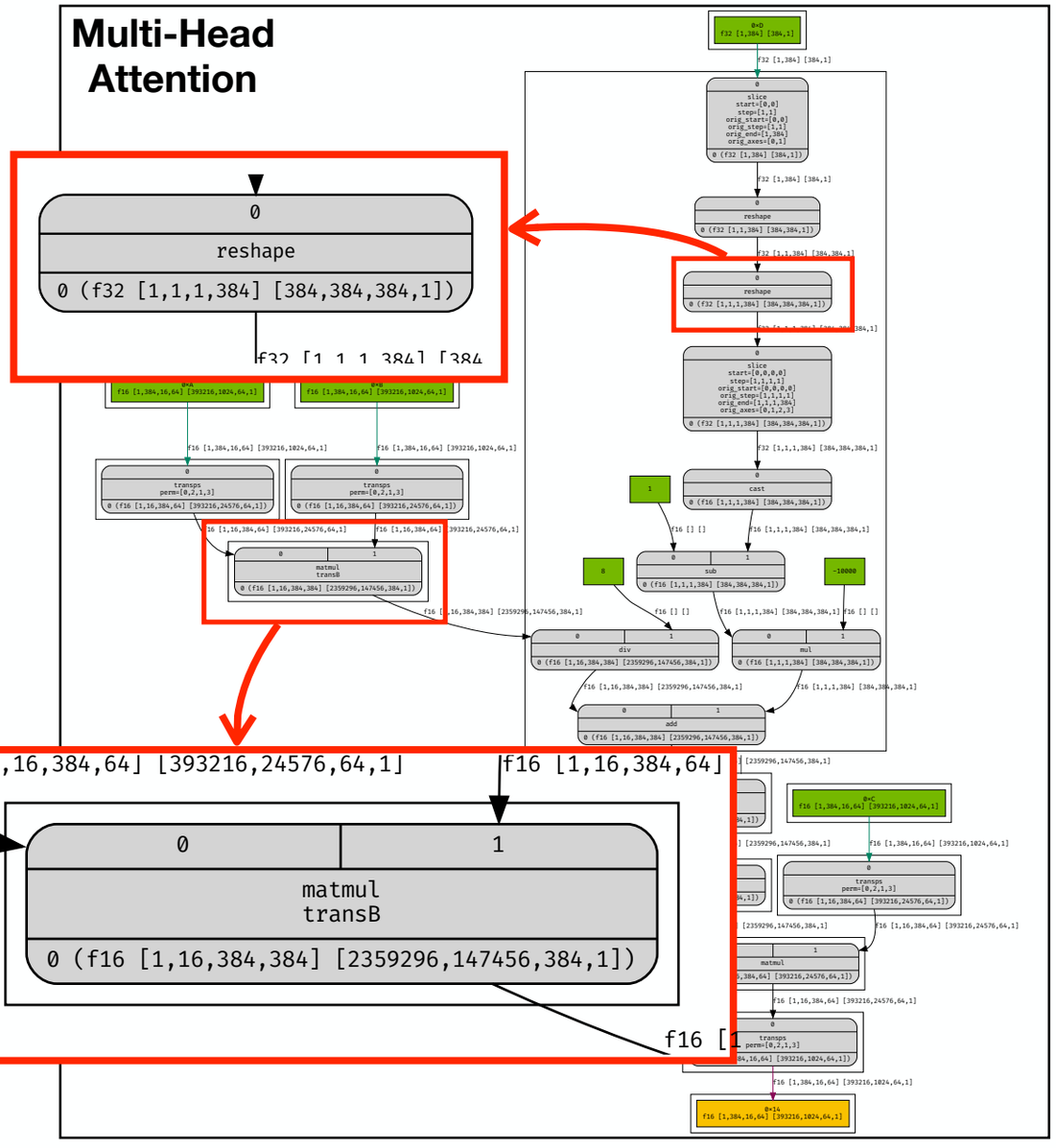
... (etc)

We expect MDH to be a promising (formal) foundation for these goals, e.g., due to its uniform representation and its captured algebraic information

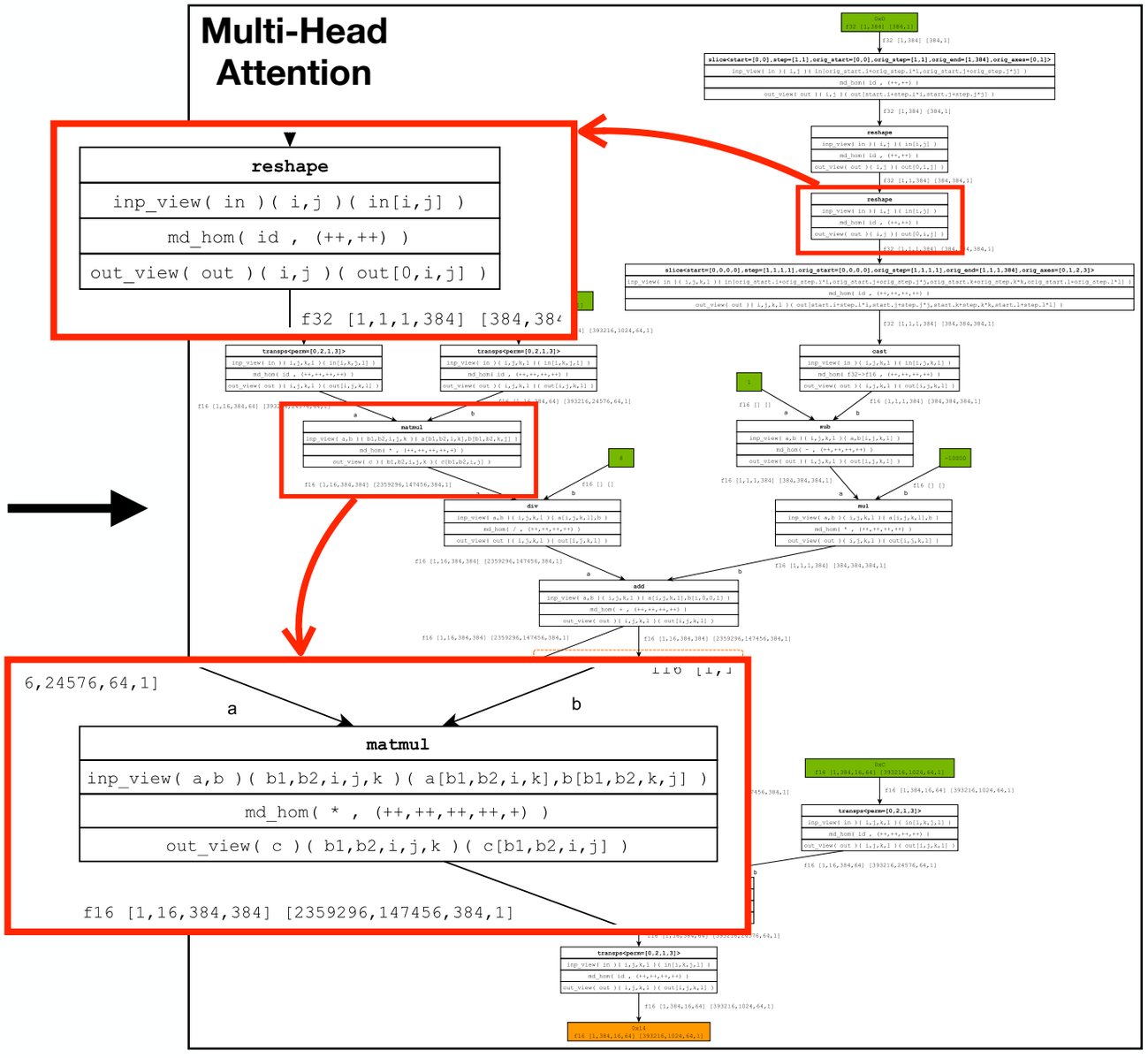
MDH: WIP & Future Directions

MDH's future goals particularly promising for DL:

Standard DL Graph



MDH DL Graph



DL Graphs can be uniformly expressed in MDH
(to benefit from MDH advantages: PPP, Fusing, …)

MDH: Summary

- MDH combines three key goals — ***Performance*** & ***Portability*** & ***Productivity*** — as compared to related approaches

- For this, MDH **formally introduces program representations** on both:

  - **high level**, for conveniently expressing — in one uniform formalism — the various kinds of data-parallel computations, agnostic from hardware and optimization details, while still capturing all information relevant for generating high-performance program code

  - **low level**, which allows uniformly reasoning — in the same formalism — about optimized (de/re)-compositions of data-parallel computations for the memory and core hierarchies of contemporary parallel architectures (GPUs, CPUs, etc)

- MDH **lowers** instances in its high-level representation to device- and data-optimized instances in its low-level representation, in a **formally sound** manner, by introducing a generic search space that is based on **performance-critical parameters** & **auto-tuning**

- Our **experiments** confirm that MDH often achieves higher ***Performance*** & ***Portability*** & ***Productivity*** than popular state-of-practice approaches, including hand-optimized libraries provided by vendors

- **Many promising future directions, particularly for DL!**

Code Optimization via **ATF**

Overview | Getting Started | Code Examples | Publications | Citations | Contact

Auto-Tuning Framework (ATF)

Efficient Auto-Tuning of Parallel Programs with Constrained Tuning Parameters

Overview

The Auto-Tuning Framework (ATF) is a general-purpose auto-tuning approach: given a program that is implemented as generic in performance-critical program parameters (a.k.a. *tuning parameters*), such as sizes of tiles and numbers of threads, ATF fully automatically determines a hardware- and data-optimized configuration of such parameters.

Key Feature of ATF

A key feature of ATF is its support for *Tuning Parameter Constraints*. Parameter constraints allow auto-tuning programs whose tuning parameters have so-called *interdependencies* among them, e.g., the value of one tuning parameter has to evenly divide the value of another tuning parameter.

ATF's support for parameter constraints is important: modern parallel programs target novel parallel architectures, and such architectures typically have deep memory and core hierarchies thus requiring constraints on tuning parameters, e.g., the value of a tile size tuning parameter on an upper memory layer has to be a multiple of a tile size value on a lower memory layer.

For such parameters, ATF introduces novel concepts for *Generating* & *Storing* & *Exploring* the search spaces of constrained tuning parameters, thereby contributing to a substantially more efficient overall auto-tuning process for such parameters, as confirmed in our *Experiments*.

Generality of ATF

For wide applicability, ATF is designed as generic in:

1. The target program's Programming Language, e.g., *C/C++*, *CUDA*, *OpenMP*, or *OpenCL*. ATF offers *pre-implemented cost functions* for conveniently auto-tuning C/C++ programs, as well as CUDA and OpenCL kernels which require host code for their execution which is automatically generated and executed by ATF's pre-implemented CUDA and OpenCL cost functions. ATF also offers a pre-implemented *generic* cost function that can be used for conveniently auto-tuning programs in any other programming language different from C/C++, CUDA, and OpenCL.

https://atf-tuner.org

ACM TACO 2021

Efficient Auto-Tuning of Parallel Programs with Interdependent Tuning Parameters via Auto-Tuning Framework (ATF)

ARI RASCH and RICHARD SCHULZE, University of Muenster, Germany
MICHEL STEUWER, University of Edinburgh, United Kingdom
SERGEI GORLATCH, University of Muenster, Germany

Auto-tuning is a popular approach to program optimization: it automatically finds good configurations of a program's so-called tuning parameters whose values are crucial for achieving high performance for a particular parallel architecture and characteristics of input/output data. We present three new contributions of the Auto-Tuning Framework (ATF), which enable a key advantage in *general-purpose auto-tuning*: efficiently optimizing programs whose tuning parameters have *interdependencies* among them. We make the following contributions to the three main phases of general-purpose auto-tuning: (1) ATF *generates* the search space of interdependent tuning parameters with high performance by efficiently exploiting parameter constraints; (2) ATF *stores* such search spaces efficiently in memory, based on a novel chain-of-trees search space structure; (3) ATF *explores* these search spaces faster, by employing a multi-dimensional search strategy on its chain-of-trees search space representation. Our experiments demonstrate that, compared to the state-of-the-art, general-purpose auto-tuning frameworks, ATF substantially improves generating, storing, and exploring the search space of interdependent tuning parameters, thereby enabling an efficient overall auto-tuning process for important applications from popular domains, including stencil computations, linear algebra routines, quantum chemistry computations, and data mining algorithms.

CCS Concepts: • **General and reference → Performance**; • **Computer systems organization → Parallel architectures**; • **Software and its engineering → Parallel programming languages**;

Additional Key Words and Phrases: Auto-tuning, parallel programs, interdependent tuning parameters

ACM Reference format:
Ari Rasch, Richard Schulze, Michel Steuwer, and Sergei Gorlatch. 2021. Efficient Auto-Tuning of Parallel Programs with Interdependent Tuning Parameters via Auto-Tuning Framework (ATF). *ACM Trans. Archit. Code Optim.* 18, 1, Article 1 (January 2021), 26 pages.
https://doi.org/10.1145/3427093

This is a new paper, not an extension of a conference paper.
Authors' addresses: A. Rasch, R. Schulze, and S. Gorlatch, University of Muenster, Muenster, Germany; emails: {a.rasch, r.schulze, gorlatch}@uni-muenster.de; M. Steuwer, University of Edinburgh, Edinburgh, United Kingdom; email: michel.steuwer@glasgow.ac.uk.

1544-3566/2021/01-ART1

Goal of **ATF**

Advantage of *Auto-Tuning Framework (ATF)* over state-of-the-art general-purpose AT approaches:

ATF finds values of performance-critical parameters with

***interdependencies* among them**

via optimized processes to

generating & *storing* & *exploring*

the spaces of interdependent parameters

For this, ATF introduces:

```
tuner.addParameter( “tp_1”, T1 );
tuner.addParameter( “tp_2", T2 );
// ...

tuner.addConstraint(
  [](T1 tp_1, T2 tp_2, … ) -> bool
```

traditional constraints

Defined on:
***search space* (traditional)**
vs. *parameters* (ATF)

```
tuner.addParameter( “tp_1”, R1, [](T1 tp_1) -> bool { /* … */ } );
tuner.addParameter( “tp_2”, R2, [](T2 tp_2) -> bool { /* … */ } );
```

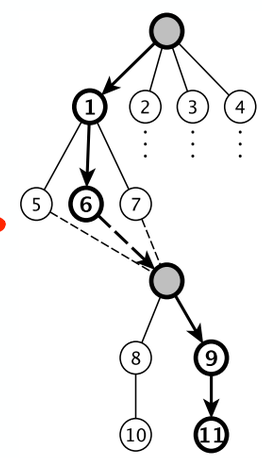
ATF
parameter constraints **[1]**

```
SP := [ (1,1) | (2,1) | (2,2) | … ]
```

traditional search space

Structure is:
***verbose & 1D* (traditional)**
vs. *compact & nD* (ATF)

ATF
CoT search space **[1]**



[1] “Efficient Auto-Tuning of Parallel Programs with Interdependent Tuning Parameters via Auto-Tuning Framework (ATF)”, *TACO’21*

ATF: User Interfaces

In a nutshell

ATF's ***Python-based*** user interface[1]:

ATF Website

Name

Range
(either `interval` or `set`)

Constraint
(may contain tuning parameters)

Arbitrary & pre-implemented *cost functions*

Various pre-implemented
Search Techniques & Abort Conditions

```python
# Input Size
N = 1000

# Step 1: Generate the Search Space
WPT = TP( 'WPT'                                ,
          Interval( 1,N )                      ,
          lambda WPT: N % WPT == 0 )
LS  = TP( 'LS'                                 ,
          Interval( 1,N )                      ,
          lambda WPT,LS: (N/WPT) % LS == 0 )

# Step 2: Implement a Cost Function
saxpy_code = # ...

N = np.int32( N )
a = np.float32( np.random.random() )
x = np.random.rand(N).astype(np.float32)
y = np.random.rand(N).astype(np.float32)

cf = opencl.CostFunction( saxpy_code )
          .platform_id( 0 )                    \
          .device_id( 0 )                      \
          .kernel_args( N, a,x,y )             \
          .glb_size( lambda WPT,LS: N/WPT ) \
          .lcl_size( lambda     LS: LS    )

# Step 3: Explore the Search Space
config = Tuner().tuning_parameters( WPT,LS )     \
                .search_technique( AUCBandit() ) \
                .tune( cf, Evaluations(50) )
```

Schulze, Gorlatch, Rasch, "pyATF: Constraint-Based Auto-Tuning in Python", CC'25

[1] ATF also offers a GPL-based interface for (online-tuning) C++ programs, as well as a DSL-based interface (offline tuning)

ATF: Summary

ATF efficiently handles tuning parameters with ***interdependencies*** among them:

ATF introduces novel concepts to
Generating & ***Storing*** & ***Exploring***
the search spaces of *interdependent parameters*, based on its *novel*
constraint design and ***search space representation***

Further ATF features (not presented on slides for brevity):

- ATF has a DSL-based **user interface** that is **arguably simpler** to use and more expressive than existing auto-tuners (including: OpenTuner & CLTune)
- ATF offers different kinds of **search techniques** and **abort conditions** (extensible)
- ATF offers a **DSL-based** user interface (*offline tuning*), as well as **GPL-based** interfaces (*online* auto-tuning):

pyATF

github.com/atf-tuner/pyATF

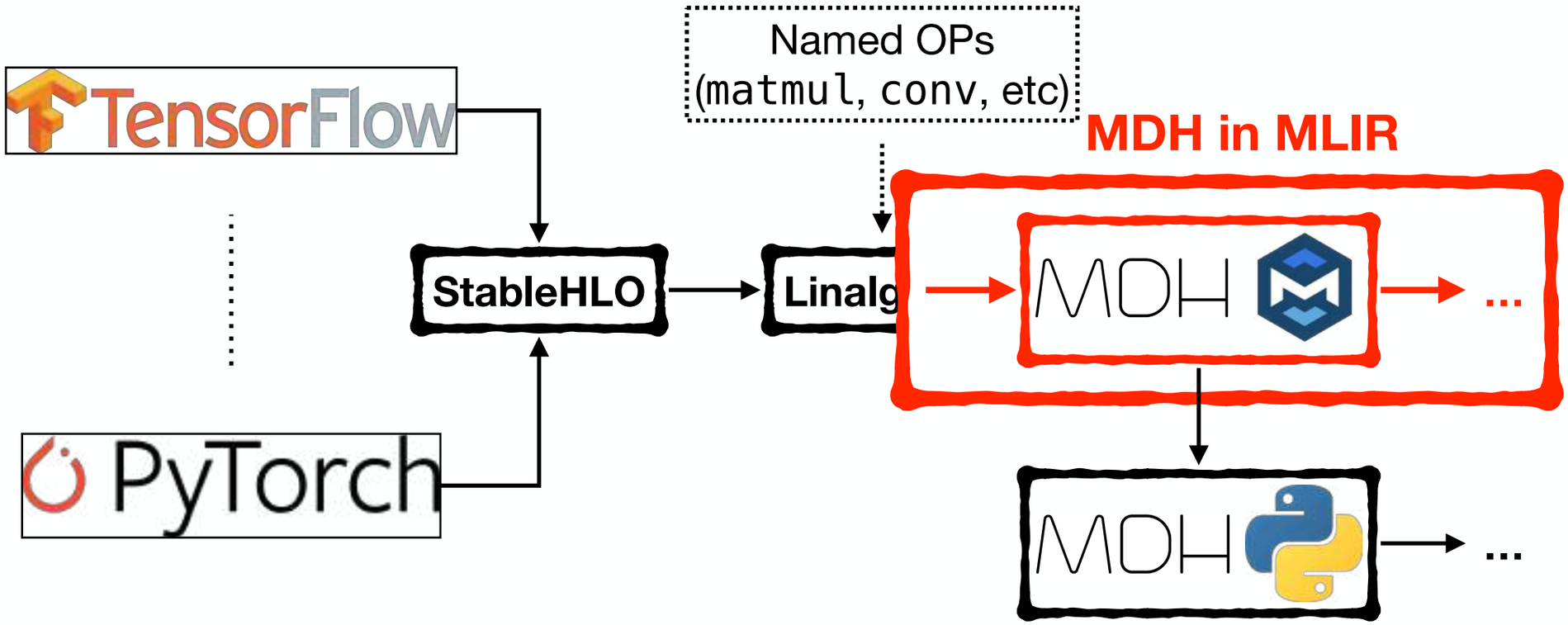
cppATF

github.com/atf-tuner/cppATF

… (future work)

roofline — Possible Collaboration(s)

Making the MLIR (sub-)project of MDH a roofline project?



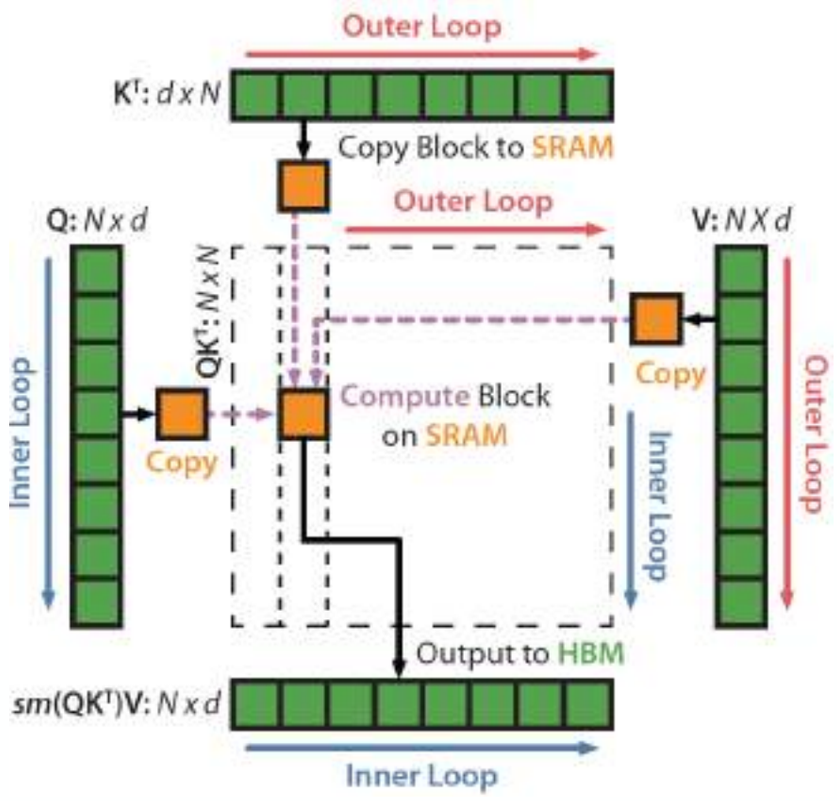
Implemented by
Jens & Lars Hunloh

“MDH in MLIR” is promising,
and there is still much work ahead!

roofline — Possible Collaboration(s)

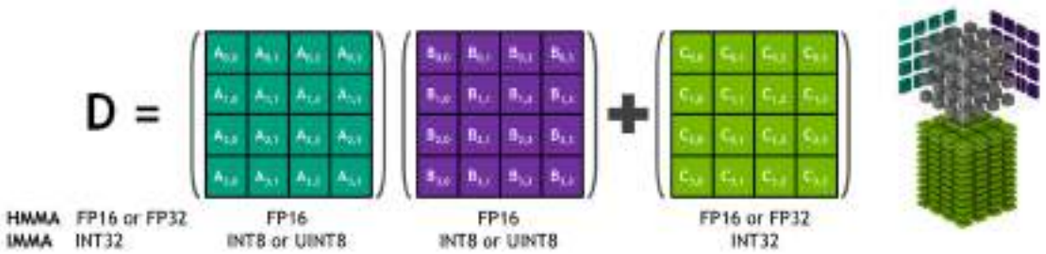
Further collaboration possibilities (that also can be combined with MLIR):

MDH-Based Code Generation for *FlashAttention*



Dao et al., “FlashAttention: Fast and Memory-Efficient Exact Attention with IO-Awareness”

MDH-Based Code Generation for *Tensor Contractions*



©NVIDIA

...

There are many further (promising) collaboration points

Universität Münster

Questions?

https://arirasch.net
a.rasch@uni-muenster.de

Ari Rasch

https://mdh-lang.org

Code Generation

https://atf-tuner.org

Code Optimization

https://richardschulze.net
r.schulze@uni-muenster.de

Richard Schulze